FMDSM Absorption Analysis Algorithm

Attenuation of a seismic wave propagating in soils is the decay of the wave amplitude in space. Total attenuation arises from geometric spreading (due to the change in wave front), apparent attenuation (due to mode conversion, reflection-refraction at an interface) and material losses (intrinsic attenuation or absorption). If you were to consider just geometric spreading and absorption, the signal amplitude A within a homogeneous medium at distance x from the source is related to the amplitude A_0 at distance x_0 by

$$A(x) = A_0 \left(x_0 / x\right)^n e^{-\alpha(x - x_0)} \tag{1}$$

Where α is the absorption coefficient. The amplitude decay term in eq. (1) $(x_0/x)^n$ corresponds to geometric spreading and based upon the conservation of the energy flux of a traveling seismic wave and spherical divergence n should equal 1, but some researchers have noted that the amplitude of the seismic wave generally does not decay as $1/r$. For that reason the exponent n is incorporated into eq. (1). The estimation of exponent n has been facilitated in the absorption estimation algorithm outlined below.

The apparent attenuation due refraction at an interface is quantified by the transmission coefficient as was outlined in Technical Note 7. The transmission coefficient quantifies the loss of energy when transitioning from layer 1 to layer 2 is outlined below in eq. (2).

$$T_{12} = \frac{\boldsymbol{A_2}}{\boldsymbol{A_1}} = \frac{2G_1\eta_1}{G_1\eta_1 + G_2\eta_2} = \frac{2Z_1}{Z_1 + Z_2} = \frac{\boldsymbol{2\rho_1 V_1 cos\theta_1}}{\boldsymbol{\rho_1 V_1 cos\theta_1 + \rho_2 V_2 cos\theta_2}} \tag{2}$$

In eq. (2) T_{12} is the transmission coefficient for the source traveling moving from layer 1 to 2, A_1 is the amplitude of incident wave, and A_2 the amplitude of refracted wave, ρ_i is the medium density of layer i, θ_1 denotes the incident angle, θ_2 is the refraction angle, V_i is the medium velocity of layer i.

The signal amplitudes for seismic traces recorded at various depths can then be defined utilizing eqs. (1) and (2) and the parameters illustrated in Fig. 1. In the subsequently outlined equations it assumed that there is an initial reference distance from the source of 0.1 m (i.e., $x_0 = 0.1m$ in eq. (1)). Equations (3), (4), (5), and (6) outlined below define the expected source amplitude for traces recorded at depths D_1, D_2, D_3, and D_4, respectively.

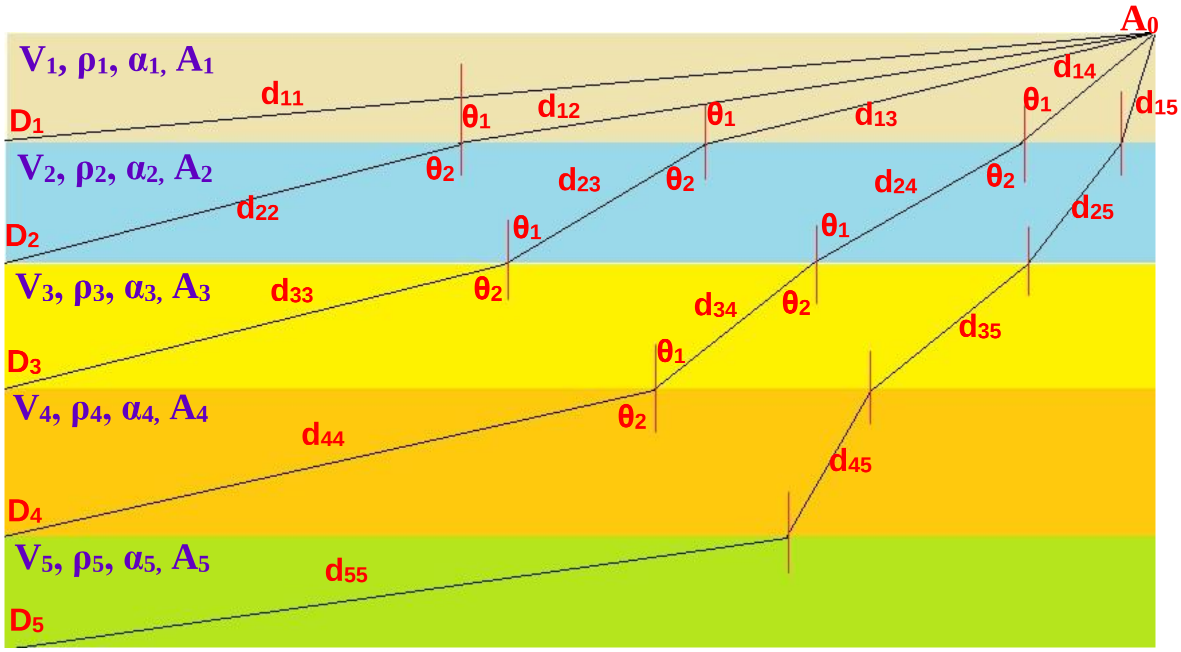

Figure 1. Cross-section of DST illustrating refracted source waves for five depth increments and required parameters for implementation of FMDSM absorption estimation algorithm.

$$A_1 = A_0\left(\frac{x_0}{d_{11}}\right)^n e^{-\alpha_1(d_{11}-x_0)} \tag{3}$$

$$A_2 = T_{12}A_0\left(\frac{x_0}{d_{12}+d_{22}}\right)^n e^{-(\alpha_1(d_{12}-x_0)+\alpha_2 d_{22})} \tag{4}$$

$$A_3 = T_{23}T_{13}A_0\left(\frac{x_0}{d_{13}+d_{23}+d_{33}}\right)^n e^{-(\alpha_1(d_{13}-x_0)+\alpha_2 d_{23}+\alpha_3 d_{33})} \tag{5}$$

$$A_4 = T_{34}T_{24}T_{14}A_0\left(\frac{x_0}{d_{14}+d_{24}+d_{34}+d_{44}}\right)^n e^{-(\alpha_1(d_{14}-x_0)+\alpha_2 d_{24}+\alpha_3 d_{34}+\alpha_4 d_{44})} \tag{6}$$

In general terms, the amplitudes recorded at each subsequent DST depth of acquisition are mathematically expressed as follows:

$$A_i = A_0\prod_{j=1}^{i-1}T_{ji}\left(x_0\Big/\sum_{j=1}^{i}d_{ji}\right)^n e^{-\left(\alpha_1(d_{1i}-x_0)+\sum_{j=2}^{i}\alpha_j d_{ji}\right)}, \quad j\geq 1, i>1 \tag{7}$$

If you then consider the ratio of the amplitudes, whether in absolute terms or globally normalized, the unknown amplitude A_0 drops out of the set of equations.

Based on the above the proposed FMDSM Absorption Analysis (FMDSMAA) Algorithm for estimating SH wave absorption coefficients can then be described as follows:

- Utilizing the standard interval velocity FMDSM technique, obtain estimates of V_i, T_{ij}, and d_{ij}.

- For the depth increments under analysis determine the maximum amplitudes from the recorded amplitudes for each depth increment from the X and Y axes seismic recordings as follows:

$$A_i^m = \max\left\{\sum_{i=1}^{n}\sqrt{x^2(i)+y^2(i)}\right\} \tag{8}$$

- Specify the estimated densities for each depth interval based upon the CPTU recordings or known values. Typical density values are outlined below

Table 1 Approximate Material Properties (from ASTM D7128)

Material	*P*-Wave Velocity (m/s)	*S*-Wave Velocity (m/s)	Density (kg/m³)	Acoustic Impedance
Dry sand/gravel	750[C]	200	1800	1.35×10^6
Clay	900	300	2000	1.80×10^6
Saturated sand	1500	350	2100	3.15×10^6
Saturated clay	1800	400	2200	3.96×10^6
Shale	3500	1500	2500	8.75×10^6
Sandstone	2850	1400	2100	5.99×10^6
Limestone	4000	2200	2600	10.4×10^6
Granite	6000	3500	2600	15.6×10^6

- Implement the FMDSMAA algorithm to calculate the synthesized amplitudes with eq. (7) based on assumed absorption coefficients, whereby the difference between the measured and synthesizes amplitude ratios is minimized.

$$\min_{\alpha_i}\left\{\sum_{i=1}^{q-1}\left(\left|\frac{A_i}{A_{i+1}}-\frac{A_i^m}{A_{i+1}^m}\right|\right)\right\} \tag{9}$$

  Where n is the number of layers or absorption coefficients to be estimated.

- The *a priori* property that absorption is inversely proportional to velocity is incorporated into the optimization algorithm so that the solution space is decreased.

It should be noted that when utilizing FMSDMAA it is mandatory that the same source energy output (e.g., same pendulum hammer height) is implemented throughout the seismic profile when acquiring seismic data.

FMDSM-Absorption - Test Bed Example

Table 2 below provides the working parameters for a test bed simulation of the FMDSMAA algorithm. For a seismic wave with a frequency of 100 Hz arrival times are assumed for 1-m thick soil layers down to a depth of 12 m. In addition for each soil layer values for the density and absorption coefficient are assumed.

Using FMDSM interval velocities are then calculated for each depth interval, and with the outcome the wavelength is calculated ($v = f \times \lambda$). In addition the Quality factor is calculated ($Q = \pi/(\alpha \times \lambda)$). The source wave raypath diagram shown in Figure 2 is used to calculate the incident and refraction angles (the results of which are shown in Table 4), after which the globally normalized maximum amplitude is calculated using eq. (1) to account for absorption and geometric spreading as the source wave travels within a specific layer and the terms of eq. (2) (shown in red) to account for the energy loss as the source wave transitions from one layer to the subsequent deeper layer.

Table 2. FMDSMAA Test Bed Example Parameters (*f* = 100 Hz)

	Assumed			Calculated			
Depth **[m]**	**Arrival Time** **[ms]**	**Density** **ρ** **[kg/m³]**	**Absorption** **α** **[1/m]**	**Interval Velocity** **v** **[m/s]**	**Wavelength** **λ** **[m]**	**Quality factor** **Q** **[1/** Np]	**Normalized Max. Amplitude** **[m/s²]**
1	17	1800	0.34129	131.5	1.32	7	1
2	25	1700	0.64708	97.1	0.97	5	0.45437
3	28	1600	0.162	176.3	1.76	11	0.341131
4	34	1400	0.275	142.8	1.43	8	0.235517
5	37	1500	0.05244	249.6	2.5	24	0.146707
6	41	1700	0.07431	222.5	2.23	19	0.111947
7	46	1600	0.1278	189.1	1.89	13	0.094043
8	49	1700	0.03701	303.2	3.05	28	0.061211
9	54	1800	0.10137	193.7	1.94	16	0.057829
10	58	1900	0.06177	242.2	2.42	21	0.042609
11	62	1500	0.05869	243.3	2.43	22	0.040829
12	65	1900	0.03048	322.1	3.23	32	0.027391

To validate the FMDSMAA algorithm the values shown in Table 1 for the arrival time, density, and the normalized maximum amplitudes are used as input data for the algorithm. In addition the source wave raypath diagram shown in Figure 2 was used to calculate the length and duration for each segment of the various raypaths (the results of which are shown in Table 4). With these values the Absorption was derived for each depth, after which the Quality Factor was determined as well. Table 3 outlines the output of the algorithm where there is very close agreement with the initial test bed values, which demonstrates the algorithm's correctness.

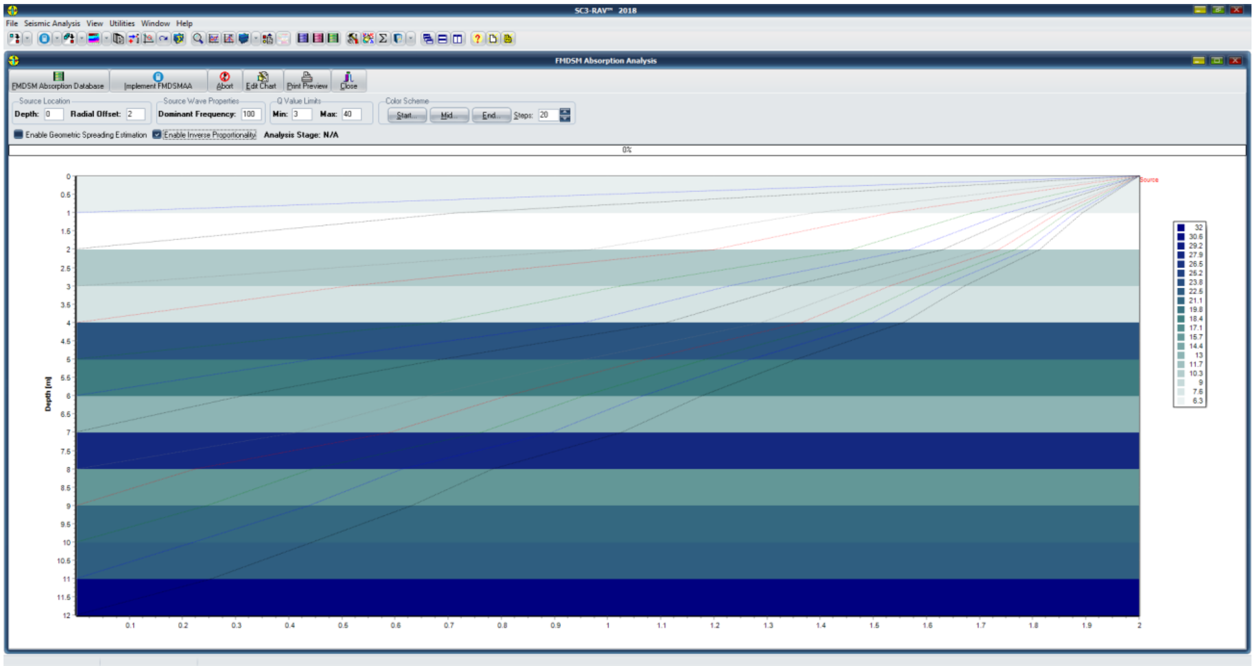

Figure 2. DST illustrating twelve simulated source waves as input into the FMDSMAA test bed.

Table 3. FMDSMAA Test Bed Example Results

Depth [m]	**Assumed Absorption** α [1/m]	**Calculated Absorption** α* [1/m]	**Q Based on α** [1/ Np]	**Q* Based on α*** [1/ Np]	**η* Damping Based on Q*** [% Np]
1	0.34129	0.36115	7	5.6	7.59
2	0.64708	0.65681	5	4.9	10.14
3	0.162	0.16951	11	10.5	4.75
4	0.275	0.27762	8	7.9	6.32
5	0.05244	0.05449	24	23.1	2.17
6	0.07431	0.07514	19	18.8	2.66
7	0.1278	0.12823	13	13.0	3.86
8	0.03701	0.0374	28	27.7	1.8
9	0.10137	0.10154	16	16	3.13
10	0.06177	0.06191	21	21	2.38
11	0.05869	0.05879	22	22	2.27
12	0.03048	0.03054	32	32	1.56

Table 4. Raypath Parameters for FMDSMAA Test Case

Ray	Distance Travelled [m]
1	2.24
2	2.86
3	3.65
4	4.5
5	5.43
6	6.36
7	7.30
8	8.27
9	9.24
10	10.22
11	11.2
12	12.18

T_C	Ray	Interface 1	Interface 2	Θ_1	Θ_2
1.041	2	1	2	51.98	35.58
1.14	3	1	2	31.42	22.63
0.86	3	2	3	22.63	44.3
1.155	4	1	2	25.02	18.2
0.806	4	2	3	18.2	34.53
1.134	4	3	4	34.53	27.34
1.168	5	1	2	17.36	12.73
0.768	5	2	3	12.73	23.57
1.155	5	3	4	23.57	18.9
0.76	5	4	5	18.9	34.48
1.172	6	1	2	13.95	10.25
0.757	6	2	3	10.25	18.84
1.161	6	3	4	18.84	15.17
0.734	6	4	5	15.17	27.21
0.982	6	5	6	27.21	24.05
1.173	7	1	2	11.97	8.8
0.752	7	2	3	8.8	16.13
1.164	7	3	4	16.13	13.01
0.723	7	4	5	13.01	23.16
0.986	7	5	6	23.16	20.52
1.102	7	6	7	20.52	17.33
1.175	8	1	2	9.63	7.1
0.747	8	2	3	7.1	12.95
1.166	8	3	4	12.95	10.46
0.713	8	4	5	10.46	18.49
0.989	8	5	6	18.49	16.42
1.105	8	6	7	16.42	13.89
0.763	8	7	8	13.89	22.65
1.176	9	1	2	8.61	6.35
0.745	9	2	3	6.35	11.57
1.167	9	3	4	11.57	9.35
0.709	9	4	5	9.35	16.48
0.99	9	5	6	16.48	14.64
1.107	9	6	7	14.64	12.39
0.758	9	7	8	12.39	20.13
1.175	9	8	9	20.13	12.7
1.176	10	1	2	7.61	5.61
0.744	10	2	3	5.61	10.21
1.168	10	3	4	10.21	8.25
0.706	10	4	5	8.25	14.5
0.991	10	5	6	14.5	12.88
1.108	10	6	7	12.88	10.9
0.754	10	7	8	10.9	17.64
1.179	10	8	9	17.64	11.16
0.868	10	9	10	11.16	13.99
1.177	11	1	2	6.83	5.03
0.742	11	2	3	5.03	9.14
1.168	11	3	4	9.14	7.38
0.704	11	4	5	7.38	12.95
0.992	11	5	6	12.95	11.5
1.108	11	6	7	11.5	9.73
0.751	11	7	8	9.73	15.69
1.182	11	8	9	15.69	9.93
0.866	11	9	10	9.93	12.44
1.116	11	10	11	12.44	12.49
1.177	12	1	2	6.04	4.45
0.742	12	2	3	4.45	8.07
1.169	12	3	4	8.07	6.51
0.702	12	4	5	6.51	11.38
0.993	12	5	6	11.38	10.08
1.109	12	6	7	10.08	8.51
0.748	12	7	8	8.51	13.67
1.185	12	8	9	13.67	8.65
0.865	12	9	10	8.65	10.79
1.115	12	10	11	10.79	10.81
0.754	12	11	12	10.81	14.36

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