Attenuation of DST SH Source Waves

The PPAs values of DST traces should generally decrease with depth. This is in contrast to DST arrival times where traces recorded at shallower depths can arrive later than deeper traces due to ray path refraction. If the attenuation of DST seismic traces was strictly dependent on absorption than it would be possible to have deeper traces with higher PPAs than shallower traces due to raypath refraction. In this case the shallower seismic trace would travel through a layer with significantly higher absorption than the deeper traveling source wave. This would require significant impedance contrast. Soils with high absorption have lower impedance compared to soils with smaller absorption.

Attenuation of a seismic wave propagating in soils is the decay of the wave amplitude in space. Total attenuation arises from geometric spreading (due to the change in wave front), apparent attenuation (due to mode conversion, reflection-refraction at an interface) and material losses (absorption). In general terms, the earth acts as both a low pass filter and an attenuator as a seismic wave travels through it. The signal amplitude A within a homogeneous medium at distance d from the source is related to the amplitude A_0 at distance d_0 by

$$A(d) = A_0(d_0/d)^n e^{-\alpha(d-d_0)} \tag{1}$$

In (1) it is assumed that the decay is due to only geometric spreading and absorption. The term $(d_0/d)^n$ defines is geometric spreading where it is typically assumed $n = 1$. The term $e^{-\alpha(d-d_0)}$ defines absorption. The *Quality Factor*, Q, is related to the absorption coefficient as follows:

$$Q = \pi/\alpha\lambda \tag{2}$$

In eq. (2) λ is the source wave's wavelength. The *Quality Factor* is a desirable term to define the absorption of a medium because it is nondispersive

The apparent SH wave attenuation due refraction at an interface is quantified by the transmission coefficient. The transmission coefficient quantifies the loss of energy when transitioning from layer 1 to layer 2 and is outlined below in eq. (3).

$$T_{12} = \frac{A_2}{A_1} = \frac{2\rho_1 V_1 cos\theta_1}{\rho_1 V_1 cos\theta_1 + \rho_2 V_2 cos\theta_2} = \frac{2Z_1 cos\theta_1}{Z_1 cos\theta_1 + Z_2 cos\theta_2} \tag{3}$$

In eq. (3) T_{12} is the transmission coefficient for the source traveling moving from layer 1 to 2, A_1 is the amplitude of incident wave, A_2 the amplitude of refracted wave, ρ_i is the medium density of layer i, θ_1 denotes the incident angle, θ_2 is the refraction angle, V_1 is the medium velocity of layer 1, and V_2 is the medium velocity layer 2 (note: $Z_i = \rho_i V_i$ is the *acoustic impedance)*. Note that the fractions of energy reflected ($E_R = R^2$ (where is the reflection coefficient)) and transmitted ($E_T = \frac{Z_2}{Z_1}T^2$) must add to one (i.e., $E_R + E_T = 1$).

As is evident from eqs. (1), (2) and (3), the attenuation of the SH DST source is dependent on geometric spreading, absorption, impedance contrast, and angles of incidence and refraction. The geometric spreading decreases the amplitude of SH source for deeper signals irrespective of raypath refraction. It just depends on the greater length of travel of deeper source waves. If $Z_2 > Z_1$ then $\theta_2 > \theta_1$ and $\alpha_1 > \alpha_2$ and vice versa. The test bed simulation (TBS) outlined below allows for a better understanding of the attenuation of SH sources waves in DST. In this near surface DST TBS there is significant impedance and absorption contrast and a relatively large source radial offset. The soil densities are assumed constant in the TBS.

Figure 1 illustrates a DST TBS with a radial source offset of 3m and significant impedance and absorption contrast of four soil layers. Figure 1 also shows the DST source wave raypaths for traces recorded at 2m, 3m, 4m and 5m. Table 1 outlines the associated source wave arrival times and interval velocities for the TBS illustrated in Fig. 1. As shown in Table 1 it is possible to have shallower traces arrive later than deeper traces due to raypath refraction. Table 1 also shows the assumed absorption values for the four soil layers. The absorption value of the relatively low impedance top layer is set at three times that of the high impedance second layer (i.e., *0.18 = 3x0.06*).

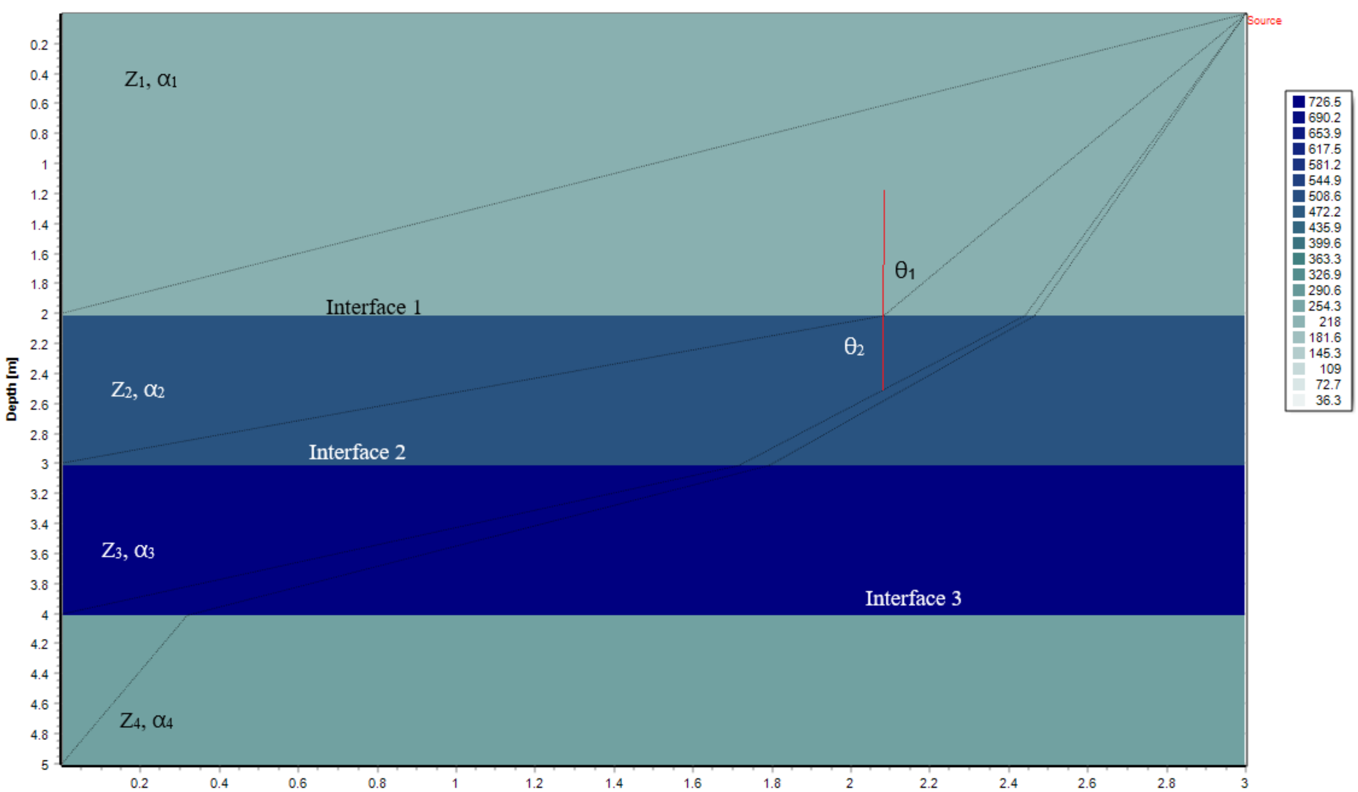

Figure 1. DTS TBS illustrating a radial source offset of 3m, significant impedance and absorption contrast of four soil layers and DST source wave raypaths for traces recorded at 2m, 3m, 4m and 5m.

Table 1. Arrival times, interval velocities and absorption values for DST TBS illustrated in Fig. 1.

Depth [m]	Arrival Time [ms]	Interval Velocity [m/s]	Q [1/N_p]	α [1/m]
2	16	225	4	0.18
3	14.5	491	20	0.06
4	14.5	727	30	0.04
5	18	269	4	0.18

Table 2 outlines the angles of incidence (θ_1) and refraction (θ_1) and corresponding transmission coefficients (eq. 3) for the source wave raypaths and soil interfaces illustrated in Fig. 1. The T values in Table 2 reflect the extent of attenuation of the source wave as it travels from the surface to the downhole receivers. As is illustrated in Table 2, it possible to have a shallower source wave more attenuated (*3m – T = 0.738*) than a deeper trace (*4m – T = 0.856*) due to impedance contrast and angles of incidence and refraction.

Table 2. Angles of incidence, angles of refraction and transmission coefficients for DST TBS illustrated in Fig. 1.

Ray	Interface 1			Interface 2			Interface 3			T
	θ_1 [°]	θ_2 [°]	T_1	θ_1 [°]	θ_2 [°]	T_2	θ_1 [°]	θ_2 [°]	T_3	$T = T_1 * T_2 * T_3$
1	56.3	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
2	24.5	64.6	0.986	N/A	N/A	N/A	N/A	N/A	N/A	0.986
3	15.6	35.9	0.706	35.9	60	1.045	N/A	N/A	N/A	0.738
4	14.9	34	0.697	34	55.8	0.999	55.8	17.28	1.229	0.856

Table 3 outlines the distances traveled for each source wave within the soil layers illustrated in Fig. 1. The geometric spreading is calculated using eq. 1 (d_0/d) and assuming d_0= *0.1m*. Parameter d_0 arbitrarily set because we concerned only with the relatively geomtetric spreading for deeper source waves. As is expected, the geometric spreading source wave attenuation increases with depth in DST.

Table 3. Source wave geometric spreading for DST TBS illustrated in Fig. 1.

Ray	d_1 [m]	d_2 [m]	d_2 [m]	d_2 [m]	d $d = d_1+d_2+d_3+d_4$ [m]	Geometric Spreading [GS]
1	3.6	N/A	N/A	N/A	3.6	0.0278
2	2.2	2.3	N/A	N/A	4.5	0.0222
3	2.09	1.23	1.98	N/A	5.3	0.01887
4	2.08	1.21	1.78	1.04	6.1	0.01639

Table 4 outlines the attenuation due to absorption for the source wave raypaths and soil interfaces illustrated in Fig. 1. These values are obtained by applying the travel distances outlined in Table 3 and the associated absorption values shown in Table 1. As is illustrated in Table 4, it possible to have a shallower source wave more attenuated (2*m – Absorption = 0.523*) than a deeper trace (*3m – Absorption = 0.586* and *4m – Absorption = 0.589*) due to raypath refraction.

Finally, the overall attenuations due to transmission, geometric spreading and absorption for the four source waves illustrated in Fig. 1 are outlined in Table 5. As is shown in Table 5, the attenuation increases with depth for the TBS DST illustrated in Fig. 1.

Table 4. Attenuation of source waves due to absorption for DST TBS illustrated in Fig. 1.

Ray	$e^{-\alpha 1(d_1)}$	$e^{-\alpha 2(d_2)}$	$e^{-\alpha 3(d_3)}$	$e^{-\alpha 4(d_4)}$	Absorption $e^1 * e^2 * e^3 * e^4$ [ABS]
1	0.523	N/A	N/A	N/A	0.523
2	0.673	0.871	N/A	N/A	0.586
3	0.686	0.929	0.924	N/A	0.589
4	0.688	0.930	0931	0.779	0.464

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Table 5. Overall attenuations due to transmission, geometric spreading and absorption for the four source waves illustrated in Fig. 1.

Transmission Coefficient T	Geometric Spreading [GS]	Absorption [ABS]	Attenuation T*GS*ABS
N/A	0.0278	0.523	0.0145
0.986	0.0222	0.586	0.0130
0.738	0.01887	0.589	0.0082
0.856	0.01639	0.464	0.0065

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