Reflection Coefficient Estimation for DST Blind Seismic Deconvolution Test Bed Simulation

A fundamental and extremely challenging seismic signal processing problem is blind seismic deconvolution (BSD), where it is attempted to deconvolve (or separate) an unknown (and possibly time variant) source wave from equally unknown in-situ reflection coefficients. Since these reflection coefficients identify and quantify the impedance mismatches between different geological layers, they are of great interest to the geotechnical and geophysical engineer. For this purpose BCE has developed a very powerful algorithm referred to as Principle Phase Decomposition (PPD™) that has been incorporated into BCE's BSDSolver™ software packages. As outlined in Technical Note 9, it is crucial that BSD is applied whenever DST data with significant source wave reflections are processed, and especially when Total Internal Reflections (TIRs) occur.

In evaluating the PPD™ algorithm extensive "Test Bed Simulation" (TBS) was carried out. For this process, which involves testing an algorithm with known and realistic input, synthetic seismograms were generated and processed. A major component of TBS for DST was to obtain accurate reflection coefficient arrival times (based upon the travel distances of the generated SH source waves) and the corresponding reflection coefficient amplitudes (based upon the impedance mismatches). This technical note outlines the methodology utilized for calculating these parameters.

Figure 1 outlines an SCPT investigation, showing the direct and reflecting source waves as well as the required parameters for calculating the source waves travel distances. This investigation is carried out between two concrete piles where the SH-wave velocity of the piles (V_2) is much greater than that of the surrounding soil (V_1). In Figure 1, ρ is the medium density, **3** denotes the first arriving direct SH source wave while **1** and **2** denote the reflecting waves, θ_1 denotes the incident and reflecting angles, θ_2 is the refraction angle, and $\beta_1 = 90° - \theta_1$.

Governing Equations

Snell's Law:

$$\frac{\sin \theta_2}{V_2} = \frac{\sin \theta_1}{V_1} = p \tag{1}$$

- If V_2 is less than V_1, then θ_2 is less than θ_1. As V_2 approaches 0 (air /ground or water/ground interface) then $\theta_2 = 0°$. In eq. (1) p is termed the *ray parameter* and denotes the horizontal slowness of the ray.

- The critical angle, Φ, is defined as the angle where $\theta_2 = 90°$ and the refracted (head / conical) wave is travelling along the interface. For angles of incidence greater than Φ, it impossible to satisfy Snell's law (using real angles) since $\sin(\theta_2)$ cannot exceed unity and internal reflections occur. This does not mean, however, that 100% of the energy is reflected: in case of an incident P-wave, a small portion of the energy is converted into S-waves and evanescent waves

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- In case of DST there are high incident angles between the reflecting piles and/or stone columns. Moreover the velocity contrast is quite large ($V_2 >> V_1$), and therefore the critical angle will be relatively small as is clear from eq. (2) which defines the critical angle:

$$\Phi = \sin^{-1}(V_1/V_2) \tag{2}$$

If we assume that $V_2 = 2V_1$ the critical angle is *30°*. As V_2 increases even more the critical angle becomes even smaller, and we can therefore safely assume that the incident angles will exceed the critical angle.

SH-Wave Reflection and Transmission Coefficients:

The SH *precritical reflection coefficients* (reflections at angles less than the critical angle) are given as

$$R = \frac{A_1}{A_0} = \frac{G_1\eta_1 - G_2\eta_2}{G_1\eta_1 + G_2\eta_2} = \frac{\rho_1 V_1 cos\theta_1 - \rho_2 V_2 cos\theta_2}{\rho_1 V_1 cos\theta_1 + \rho_2 V_2 cos\theta_2} \tag{3}$$

$$T = \frac{A_2}{A_0} = \frac{2G_1\eta_1}{G_1\eta_1 + G_2\eta_2} = \frac{2\rho_1 V_1 cos\theta_1}{\rho_1 V_1 cos\theta_1 + \rho_2 V_2 cos\theta_2} \tag{4}$$

where R is the reflection coefficient, T the transmission coefficient, A_0 the amplitude of incident wave, A_1 the amplitude of reflected wave, A_2 the amplitude of refracted wave (note: $Z_i = \rho_i V_i$ is the *acoustic impedance)*, G_i the shear modulus of medium i (note $G = \rho V_s^2$), and η_i the vertical slowness within medium i. The latter can be given as:

$$\eta_1 = u_1\, cos\theta_1 = cos\theta_1/V_1 = \sqrt{u_1^2 - p_1^2} \tag{5a}$$
$$\eta_2 = u_2\, cos\theta_2 = cos\theta_2/V_2 = \sqrt{u_2^2 - p_2^2} \tag{5b}$$

To meet boundary conditions (i.e., abide by Snell's Law) it is required that $p = p_1 = p_2$ in eqs. (5a) and (5b).

At the *critical angle*, which as mentioned earlier is defined as the angle where $\theta_2 = 90°$, sin $\theta_2 = 1$ and cos $\theta_2 = 0$. Therefore using eqs. (1), (3), (4) and (5b):

- the ray parameter $p = u_2$
- the reflection coefficient $R = 1$
- the transmission coefficient $T = 2$
- the slowness in medium 2 $\eta_2 = 0$

This means that the transmitted SH wave has an amplitude equal to twice that of the incident wave and travels along the interface (also termed a Head Wave), while the reflected wave has the same amplitude as the incident wave, and there is no vertical transmission[1].

At the *post-critical angle* θ_2 becomes complex. This becomes evident when eq. (1) is rewritten as

$$\theta_2 = \sin^{-1}\left(\frac{v_2}{v_1}\sin\theta_1\right) \tag{6}$$

Since $v_1 < v_2$ the arcsin function can become greater than 1, which results in a complex solution for θ_2. Consequently η_2 is imaginary, and transmitted waves with imaginary slowness are termed *evanescent waves.* Evanescent waves have no vertical energy flux where the energy normalized transmission coefficient is zero and their amplitude decays exponentially with depth.

Since any incident angle greater than the critical angle will be largely reflected from the boundary instead of being refracted, *post-critical angle* reflections are referred to as TIRs due to the fact that a negligible amount of energy is transmitted and nearly all of the energy reflected. In Technical note 8 the corresponding source wave distortions which occur due to the multiplication of the source wave with complex reflection coefficients will be addressed.

Trigonometric Relationships (see Figure 1):

$$d_1 = \frac{l_1}{tan\beta_1} \tag{7}$$

$$d_2 = \frac{l_2}{tan\beta_1} \tag{8}$$

$$\theta_1 = 90° - \beta_1 \tag{9}$$

$$d = d_1 + d_2 \tag{10}$$

$$d\tan\beta_1 = l_1 + l_2 \quad (11$$

$$\beta_1 = \tan^{-1}(l_1 + l_2\,/d) \tag{12}$$

$$SD1 = \sqrt{d_1^2 + l_1^2} \tag{13}$$

$$SD2 = \sqrt{d_2^2 + l_2^2} \tag{14}$$

Travel Distance for Source Wave 1: $$SD = SD1 + SD2 \tag{15}$$

Travel Time for Source Wave 1: $$t = SD/V_1 \tag{16}$$

In eqs. (7) to (14) the known parameters are d (probe depth), l_1 concrete pile-source radial offset, and l_2 concrete pile-seismic probe radial offset.

[1] Continuity of displacement requires $A_0 + A_1 = A_2$. The sum of the energy flux density on the interface from all of the scattered waves must equal the energy flux density from the incident waves.

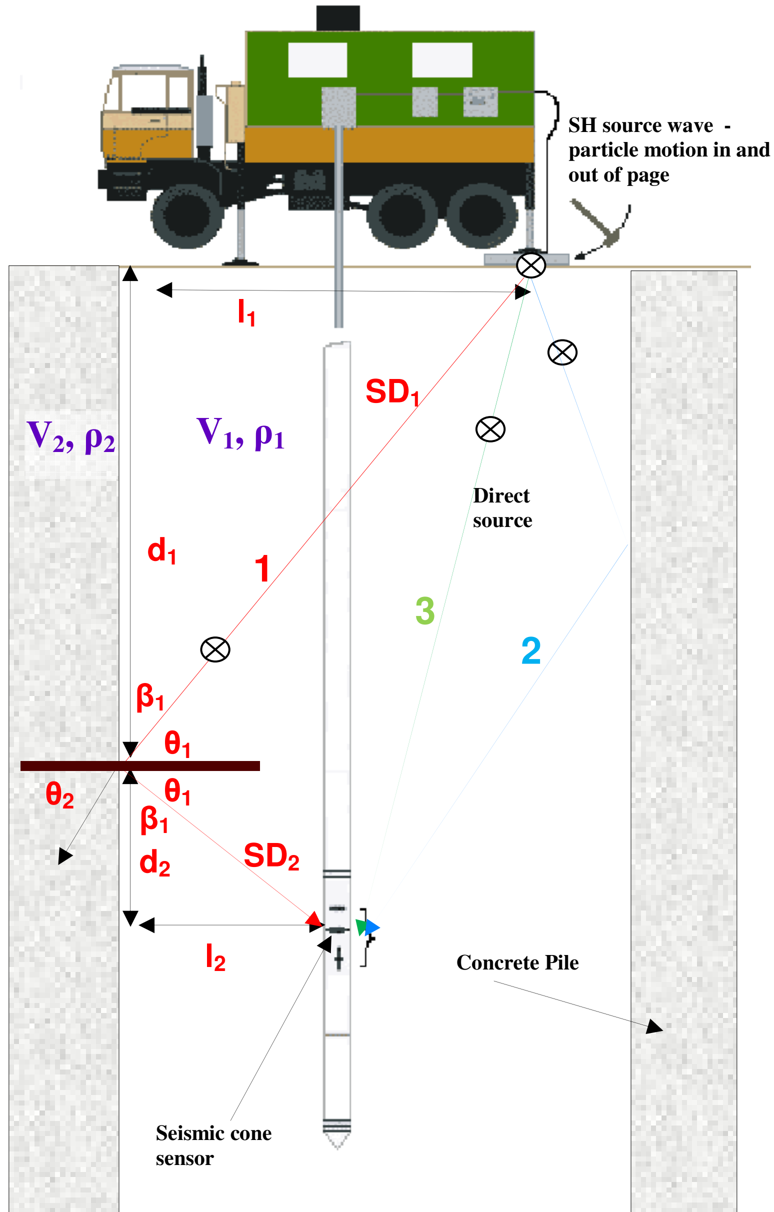

Figure 1. Cross-section of SCPT illustrating direct and reflecting source waves and required parameters for calculating the source waves travel distances.

<u>TBS Simulation Methodology</u>

The methodology for obtaining the reflection coefficients' arrival time and amplitude is then as follows:

Reflection Coefficient Arrival Time:

- Specify d, l_1, and l_2.
- Use eq. (12) to calculate β_1.
- Calculate d_1 and d_2 using eqs. (7) and (8), respectively.
- Calculate SD_1 via eq. (13).
- Calculate SD_2 via eq. (14).
- Apply eqs. (15) and (16) to obtain the reflection coefficient arrival time.

Reflection Coefficient Amplitude:

Use eq. (3) to calculate the amplitude of the reflection coefficient for angles of incidence less than Φ.

As stated previously, for angles of incidence greater than Φ, it impossible to satisfy Snell's law (using real angles) and total reflection occurs. This does not mean that 100% of the energy is reflected; therefore, for this case we define the reflection coefficient amplitude to be *0.7-0.9* (i.e., $A_1 = 70\%$ to *90%* of A_0) to take into account the generation of evanescent waves and geometric spreading due to the relatively longer travel path compared to the direct wave. For TBS of BSD the reflection coefficient for the direct wave is set at 1.0 and all subsequent reflection coefficient are set relative to the direct wave. This is due to the fact that the amplitude of the direct is nearly always the strongest compared to the reflected source waves.

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