Dead Reckoning / Target Tracking in an ECDIS Environment

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BIOGRAPHY

The author was born in Los Angeles, California, on December 7, 1963. He attended the University of British Columbia, where he received his B.A.Sc degree in 1986, the M.A.Sc. degree in 1988, and carried out a year of post graduate studies in electrical engineering.

Erick Baziw's interest includes the application of Kalman Filtering and Time Series Analysis to the modelling, estimation and control of dynamic systems. He has published two technical papers in this area. Erick is a senior software engineer at Offshore Systems Ltd.

ABSTRACT

Since 1979, Offshore Systems Ltd. (OSL) has been involved in the development of Electronic Chart Display and Information Systems (ECDIS) and has recently incorporated radar overlay into ECPINS[2], thus enabling the implementation of target-tracking filters. The introduction of radar overlay and Kalman Filters into ECDIS development will facilitate the introduction of a single-person bridge in which navigation equipment will be housed in a single unit and electronic charts will replace the use of paper charts. The target tracking and dead reckoning filters will also contribute towards automating mariner navigation, thus making it more reliable, accurate, and safe by making it less dependent on the operator. This paper presents and analyzes a Kalman Filter which is designed for vessel navigation and target tracking.

To reduce computer computation, the states in the optimal navigation and target-tracking system model were decoupled, resulting in two cascaded Kalman Filters. The first filter outlined is an eleven state extended dead reckoning Kalman filter (DRKF). The second filter outlined is a twelve state extended target tracking Kalman filter (TTKF). This paper discusses these two filters in detail, and outlines their state and measurement model equations. In addition, the paper illustrates how these two filters are integrated so that a single DRKF feeds vessel position and velocity estimates to multiple twelve-state TTKF filters in order to track multiple targets.

The performance of the dead reckoning / target tracking Kalman filter was evaluated by processing synthetic data. The true and estimated position and velocity states are presented and compared along with the corresponding estimation error covariance values. From the results obtained and analyses conducted, it was found that the decoupled system model and serial linked Kalman Filters performed close to the coupled system and that a substantial savings of computer computation was achieved.

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[2] Electronic Chart Precise Integrated Navigation System

1.0 INTRODUCTION

ECDIS is a fairly new technology in the marine industry and is slowly starting to take root. OSL has been involved in ECDIS development for the past 14 years and has gained extensive insight into its capabilities and limitations [1]. The highly attractive feature of ECDIS is that it combines hydrographic chart data with position and orientation information obtained from the ship's gyrocompass and electronic navigation aids and it displays the information on a video screen [2]. Several navigational measurements are integrated in order to provide the mariner with real-time tracking of the vessel's position relative to coastal and bathymetric landforms and possible traffic.

This paper outlines OSL's current development of filters which allow for this real-time tracking of the vessel and radar tracked targets. The kinematic and measurement data is designed to fit into a Kalman Filter (KF) formulation. The Kalman Filter is an optimal (in a least squares sense) recursive filter which is based on state-space, time-domain formulation of physical problems. A thorough discussion of the continuous, discrete and nonlinear versions of the KF is given in [3]. Several of the marine navigational measurement aids are nonlinear in nature (e.g., radar, gyrocompass, and speedlog); therefore, it is necessary to apply a set of KF equations which takes into account these nonlinearities.

There are several creative KF variations for taking into account nonlinear dynamics, but the two most widely used are the Extended and Linearized Kalman Filter (EKF and LKF) and their derivatives. The LKF is so designed that it linearizes about some nominal conditions in state space, while the EKF linearizes about the state space that is continually updated with the state estimates resulting from measurements. The EKF is selected rather than the LKF, because in marine navigation one is not linearizing about a nominal set of states. The basic development of the EKF is outlined in [4]. The following sections discuss the Dead Reckoning Kalman Filter (DRKF) and Target Tracking Kalman Filter (TTKF). The TTKF applies the vessel's position and velocity estimates from the DRKF and radar range and azimuth measurements in order to derive target position and velocity estimates. The DRKF and TTKF are discussed in detail in Sections 2.0 and 2.2.

In general, the number of multiplications necessary in a standard error covariance calculation varies as the third power of the number of states [5]. Therefore, the system model and corresponding state size of the optimal dead reckoning/target tracking Kalman filter (DRTTKF) was reduced. The optimal DRTTKF would have consisted of a single system model describing the vessel and target kinematic and measurement equations and containing error covariance matrix dimensions of up to 131x131 (i.e., for twenty targets). The decoupling of the system states in the optimal DRTTKF was performed to reduce the amount of numerical operations necessary to compute the error covariance matrix to a practical amount.

The computational saving in decoupling the DRTTKF system state size into one eleven state DRKF and multiple twelve state TTKFs can be substantial. If one considers the third power rule, an n-state optimal DRTTKF would require kn^3 (k reflects the computer processing speed) multiplications each time its covariance matrix is propagated. If one breaks the n state optimal DRTTKFs, each with dimension n/m, the number of multiplications to propagate the error covariance matrix is $mk(n^3/m^3)$, which is a substantial computation reduction. For example, if one considers a suboptimal DRTTKF with 10 targets, one has $10k(n^3/1000)$ or $0.01kn^3$ multiplications; this results in approximately a 99% reduction in computer calculations. Sections 2.0 and 2.2 discuss the suboptimal DRTTKF in more detail and Sections 2.1 and 2.3 analyze the performance of the suboptimal DRTTKF when processing synthetic data.

2.0 Dead Reckoning Kalman Filter

To get a current position estimate, dead reckoning requires obtaining good estimates of the velocity vector in order to integrate it ahead in time from some known position.

The filter presented in this section attempts to integrate all of the vessel's instrumention to get the most accurate vessel position and velocity estimates. In addition, the Kalman Filter introduced here takes into account vessel acceleration and ocean current velocity, which are modeled as first order Gauss-Markov processes.

Gauss-Markov Process

To have more realistic vessel dynamics, the system model includes an acceleration state, a(t), which is modeled as a first order Gauss-Markov process driven by white noise. In addition, the ocean current, Loran C, and speedlog measurement errors are also modelled as Gauss-Markov processes [6].

A Gauss-Markov process is generated by passing white noise N(0,1)[3] through a linear system transfer function $(2\sigma^2\beta)/(s+\beta)$; thus, for the continuous system, one has

[3]The notation N(0,1) denotes a gaussian random variable with mean value 0 and variance 1.

$$\dot{a}(t) = -\beta a(t) + \sqrt{2\sigma^2\beta}\,w(t), \text{ with}$$
$$E[w(t)w(\tau)] = \delta(t-\tau),\ T_c = \frac{1}{\beta} \text{ is the} \quad (1)$$
$$\text{time constant (1/sec) and } \sigma^2 \text{ is the variance}$$

To obtain the discrete form for equation (1), a sampling interval Δ was assumed and then (1) was solved over this interval. Since β is a constant, the state transition function is given by

$$\Phi = \Phi(k+1,k) = \exp^{-\beta\Delta} \quad (2a)$$

The input transition function can be determined by calculating the mean-square response of a(t),

$$\Gamma^2 = 2\sigma^2\beta\int_0^\Delta\int_0^\Delta \exp^{-\beta u}\exp^{-\beta v}\delta(u-v)\,du\,dv = \sigma^2(1-\exp^{-2\beta\Delta}) \quad (2b)$$

Thus the discrete model for the Gauss-Markov process can be written as

$$a(k+1) = a_w a(k) + b_w w_a(k) \quad \text{where, } a_w = \Phi, \text{ and } b_w = \Gamma \quad (3)$$

In equation (3), $w_a(k)$ is a zero-mean, timewise-uncorrelated, unit-variance sequence with a Gaussian probability distribution function. The Gauss-Markov process, a(t), is therefore a zero-mean, exponentially-correlated random variable whose standard deviation is σ. The constant Φ can have a range of values from 0 to +1 for $\beta \geq 0$. For $\Phi \to 0$, a(t) changes rapidly and tends to be uncorrelated from sample to sample. For $\Phi \to 1$ the behaviour of a(t) becomes more sluggish and it tends to change little from sample to sample [7].

System Model

To specify the system equations in the standard KF form, the following states need to be defined

States to be estimated (4a)

x_1 = *vessel x position*
x_2 = *current velocity in x direction*
x_3 = *vessel velocity in x direction*
x_4 = *vessel acceleration in x direction*
x_5 = *vessel y position*
x_6 = *current velocity in y direction*
x_7 = *vessel velocity in y direction*
x_8 = *vessel acceleration in y direction*
x_9 = *Loran C error in x direction*
x_{10} = *Loran C error in y direction*
x_{11} = *Speedlog error*

The continuous system equation for the KF formulation is defined in the following standard form:

$$\dot{\underline{x}} = F\underline{x} + G\underline{w} \quad (4b)$$

Therefore, with the states as defined in equation (4a), the following state matrix, F, was derived together with the input matrix, G, for the continuous system model specified by equation (4b):

$$F = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\beta_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\beta_v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{lcx} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{lcy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{sl} \end{bmatrix} \quad (4c)$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_{lcx} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \alpha_{lcy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_{sl} \end{bmatrix} \quad (4d)$$

where β_c, $\alpha_c = \sqrt{2\sigma_c^2\beta_c}$, β_v, $\alpha_v = \sqrt{2\sigma_v^2\beta_v}$, β_{lcxy}, $\alpha_{lcxy} = \sqrt{2\sigma_{lcxy}^2\beta_{lcxy}}$, β_{sl}, *and* $\alpha_{sl} = \sqrt{2\sigma_{sl}^2\beta_{sl}}$, *are defined by equation* (1) *for current, vessel, Loran C, and speedlog, respectively.*

The discrete transition matrix, Φ, of the state estimate extrapolation equation then becomes

$$\Phi = \begin{bmatrix} 1 & \Delta t & \Delta t & \Delta t^2/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t & \Delta t & \Delta t^2/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \Delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_v & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{lcx} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{lcy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{sl} \end{bmatrix} \quad (5)$$

where a_c, a_v, a_{lc}, *and* a_{sl} *are defined by equation* (2) *for the current, vessel, Loran C, and speedlog state variables, respectively.*

The discrete covariance structure, Q_k, of the input sequence w(k) is calculated as follows:

$$Q_k = E[\underline{w}(k)\ \underline{w}(k)^T]$$

$$= E\left\{\left[\int_{t_k}^{t_{k+1}} \Phi(t_{k+1},u)G(u)\underline{w}(u)du\right]\left[\int_{t_k}^{t_{k+1}} \Phi(t_{k+1},v)G(v)\underline{w}(v)dv\right]^T\right\}$$

$$= \int_{t_k}^{t_{k+1}}\int_{t_k}^{t_{k+1}} \Phi(t_{k+1},u)G(u)E[\underline{w}(u)\underline{w}(v)^T]G^T(v)\Phi^T(t_{k+1},v)dudv \quad (6)$$

since w(k) *is a white noise vector process, we have*

$$E[\underline{w}(u)\underline{w}(v)^T] = I_{7x7}\delta(u-v),$$

where I_{7x7} *denotes a 7x7 identity matrix and* $\delta(u-v)$ *denotes the dirac delta function.*

Measurement System

The measurements presently incorporated into the dead reckoning filter consist of GPS position and velocity, Loran C position, speedlog velocity magnitude, and vessel bearing, i.e.,

Measurements

z_1 = *GPS x position*
z_2 = *GPS y position*
z_3 = *GPS x velocity with respect to ground*
z_4 = *GPS y velocity with respect to ground* (7a)
z_5 = *Loran C x position*
z_6 = *Loran C y position*
z_7 = *Speedlog velocity magnitude*
z_8 = *Gyroscope vessel bearing*

The speedlog and gyro measurements are nonlinear functions of the state $\underline{x}$. The linearized measurement matrix, H, is given by the following equations

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & H_{72} & H_{72} & 0 & 0 & H_{76} & H_{76} & 0 & 0 & 0 & 1 \\ 0 & H_{82} & H_{82} & 0 & 0 & -H_{86} & -H_{86} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7b)$$

$$\text{where } H_{82} = \frac{x_6 + x_7}{(x_2+x_3)^2 + (x_6+x_7)^2}$$

$$\text{and } H_{86} = \frac{x_2 + x_3}{(x_2+x_3)^2 + (x_6+x_7)^2}$$

The elements H_{72} and H_{76} describe the speedlog measurements which require special consideration.

Speedlog Measurement

The speedlog has two modes of operation, namely, "ground-lock" and "bottom-lock". Ground-lock occurs when the velocity measurements are made with respect to the ocean floor. Bottom-lock occurs when the velocity measurements are made with respect to some depth within the ocean. Thus, for these two modes of operation, one has the following measurement model:

Ground-lock

$$z_7 = \sqrt{(x_2 + x_3)^2 + (x_6 + x_7)^2} + x_{11} + v_7$$
$$H_7 = [0\ H_{72}\ H_{72}\ 0\ 0\ H_{76}\ H_{76}\ 0\ 0\ 0\ 1] \quad (7c)$$
$$\text{where } H_{72} = \frac{x_2 + x_3}{\sqrt{(x_2 + x_3)^2 + (x_6 + x_7)^2}}$$
$$\text{and } H_{76} = \frac{x_6 + x_7}{\sqrt{(x_2 + x_3)^2 + (x_6 + x_7)^2}}$$

Bottom-lock:

$$z_7 = \sqrt{x_3^2 + x_7^2} + x_{11} + v_7$$

$$H_7 = \left[0 \ 0 \ \frac{x_3}{r} \ 0 \ 0 \ 0 \ \frac{x_7}{r} \ 0 \ 0 \ 0 \ 1\right], \quad \text{where } r = \sqrt{x_3^2+x_7^2} \qquad (7d)$$

The measurement matrix previously defined requires that the dimension is adjusted dynamically depending upon current available measurements. For instance, if all measurements are available, one has a linearized measurement matrix of dimensions 8x11. If GPS and Loran C are lost, one has a dead reckoning situation, where the speedlog and bearing measurements are used to estimate the velocity vector in order to propagate the vessel position estimate ahead. In this case the linearized measurement matrix has a dimension of 2x11.

The computational sequence for the dead reckoning KF is summarized as follows:

- *At* t_o *specify* $\hat{\underline{x}}_o$ *and* P_o, *and compute* Q_o, $\Phi(1,0)$, h_1, $H_1 = \frac{\partial h_1}{\partial x}$ *and* R_1.
- *At* t_1 *compute the projected estimate of the covariance matrix* $P(1|0) = \Phi(1,0)P_o\Phi(1,0)^T+Q_o$.
- *Compute the gain matrix* $K_1 = P(1|0)H_1^T(\Phi(1|0)\hat{\underline{x}}_o)\left[H_1(\Phi(1|0)\hat{\underline{x}}_o)P(1|0)H_1^T(\Phi(1|0)\hat{\underline{x}}_o) + R_1\right]^{-1}$.
- *Using the measurement* $\underline{z}_1$ *at* $t=t_1$, *the best estimate of the state at* t_1 *is given by* $\hat{\underline{x}}_1 = \Phi(1|0)\hat{\underline{x}}_o + K_1[\underline{z}_1 - h_1(\Phi(1|0)\hat{\underline{x}}_o)]$.
- *The estimation covariance matrix at* t_1, *is given by* $P(1|1) = P(1|0) - K_1H_1(\Phi(1|0)\hat{\underline{x}}_o)P(1|0)$.
- *At* $t=t_2$, *a new measurement* z_2 *is obtained and the computational cycle is repeated.*

2.2 Dead Reckoning Kalman Filter Results

The performance of the DRKF was determined by simulating the vessel, current,and measurement kinematics with additive Markov and white noise. The analysis was performed on a 486 computer in an OS/2 environment with Presentation Manager graphical user interface. The multitasking environment of OS/2 allowed the simulator, Kalman Filters, and ECDIS to run in parallel.

The simulator allowed the user to navigate a vessel in Vancouver, B.C. Harbour area by specifying position, velocity and acceleration of the vessel. From this kinematic information, synthetic data for the GPS, Speedlog, Loran C, and bearing measurements were generated. Table I summarizes the Markov vessel acceleration and current velocity models and the measurements noise models in five simulated tests. The acceleration model is the same for all tests. The current velocity model varies for each test because it had a stronger effect on the filter performance. Table II summarizes the vessel kinematics that were simulated. The 'time' column specifies when the velocity and rotation rate were initialized.

The Universal Transverse Mercator (UTM) coordinate system (WGS-84 datum) is used, and any measurements specified in latitude/longitude are converted to their UTM equivalents. This is done so that the nonlinearity of latitude and longitude are included in the system equations. For brevity, only the UTM x coordinate estimation data are presented.

Figure 1 illustrates the estimated and true UTM x position time histories for the five tests and Figure 2 illustrates the corresponding estimation error covariance. When GPS is on, Figure 2 illustrates how the initial error decreases until a low steady state value is reached. When GPS is not available, the error increases to reflect the Loran C Markov error. For both GPS and Loran C there is very small error in the estimated x position.

The loss of GPS and Loran C reflects a dead reckoning situation where new x position estimates are obtained by integrating the estimated velocity vector. As is illustrated in Figure 2, there is a steady increase in the position error because the velocity error is propagated and integrated ahead in time. The greatest factor affecting position error is the magnitude of the velocity vector. Test 2, which has the highest initial velocities, has a relatively high velocity error which is propagated ahead in the position estimate. Another factor which influenced the position errors was the bearing rate. Even though Test 1 had the smallest initial velocity vector, its error is equivalent to that of Test 5 and greater than that of Test 3 and 4 because of Test 1's higher bearing rate. This result is also evident when comparing Tests 3 and 4.

When the speedlog is in "bottom-lock" mode, there is a larger increase in the estimation error covariance depending upon the magnitude of the current velocity variance. Test 1 and 2 indicate a noticeable increase in slope of the estimation error. Test 2 has a higher vessel velocity vector and current velocity variance corresponding to a greater error slope. In addition, the magnitude of the speedlog error also results in higher estimation errors.

Table I: Parameters Set for DRKF*

Test	Current σ_c^2 m^2/s^2	Current β_c 1/s	GPS R_1 m^2	GPS R_2 m^2	GPS R_3 m^2/s^2	GPS R_4 m^2/s^2	L-C $\sigma_{x,y}$ m^2	S-L σ_ϕ m^2/s^2
1	0.3	0.17	1600	1600	0.25	0.25	500	0.5
2	0.7	0.33	800	800	0.25	0.25	500	0.5
3	0.25	0.17	800	800	0.5	0.5	1100	0.025
4	0.0025	0.17	800	800	0.5	0.5	1100	0.0025
5	0.1	0.125	1600	1600	0.5	0.5	1100	0.025

*L-C and S-L refer to Loran C and Speedlog, respectively. $\beta_{L-C} = 0.25$ and $\beta_{S-L} = 0.5$ for all tests conducted. R_1 to R_4 are the error covariances on the GPS measurements. The bearing measurement error is 1 deg. and constant. The acceleration model is defined as $\sigma_v = 0.5$ and $\beta_v = 1$.

Table II. Vessel Kinematic Values*

Test	Vessel Velocity Kinematics: Time	Vxo m/s	Vyo m/s	Bearing Rate: Time	deg/s
1	104	5.0	5.0	235	4.58
2	150	10.0	10.0	220	2.29
3	0	6.0	6.0	0	3.44
4	0	7.0	7.0	80	4.01
5	0	-8.0	-8.0	80	1.72

*note: $V_x(k+1) = \cos(\dot{\theta}\Delta)V_x(k) + \sin(\dot{\theta}\Delta)V_y(k)$ and $V_y(k+1) = \cos(\dot{\theta}\Delta)V_y(k) - \sin(\dot{\theta}\Delta)V_x(k)$, where $\dot{\theta}$ and Δ are the bearing and sampling rates, respectively.

Figure 1. Comparing Estimated and True UTM X Values

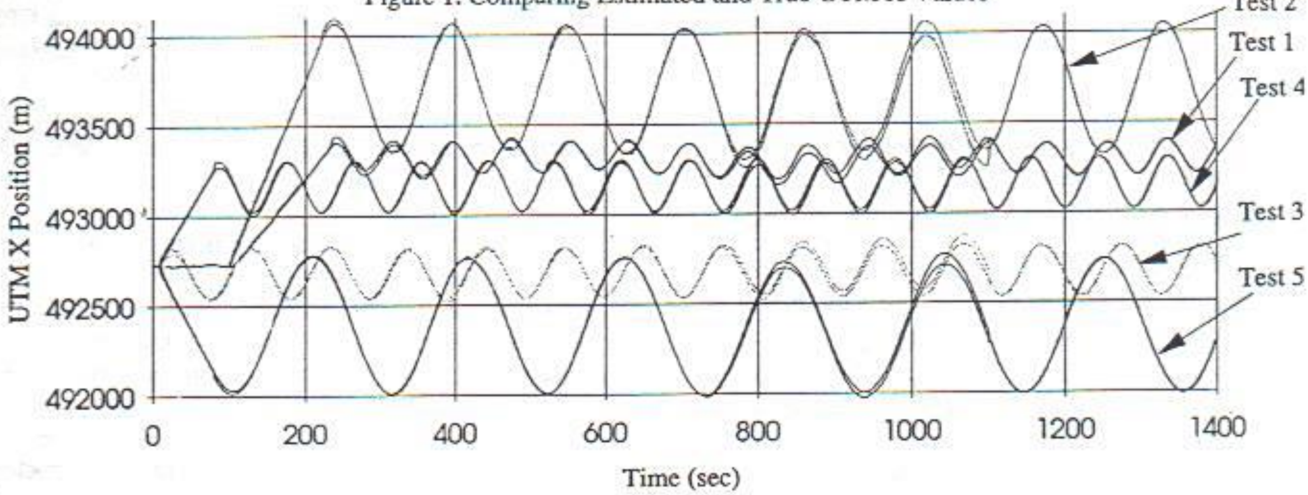
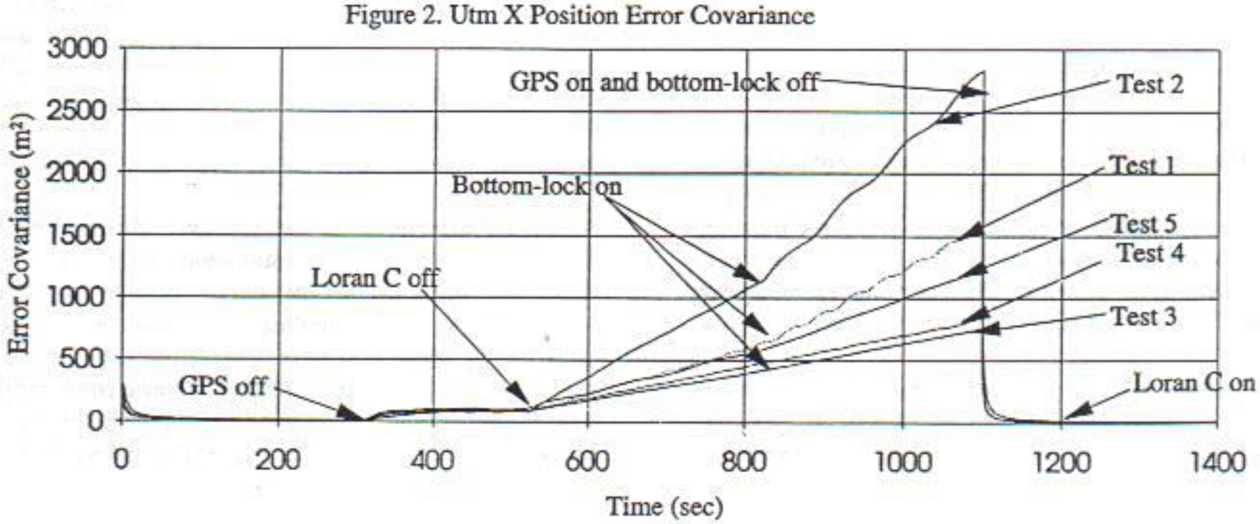


Figure 2. Utm X Position Error Covariance



Test 1 and 2 have higher speedlog Markov variance errors than test 3, 4, and 5. In all five tests there is a noticeable difference between the true and estimated UTM x positions.

When GPS and "ground-lock" are turned on, the estimation error dramatically decreases. Actually the estimation error variances give a good parameter for deciding how long one can continue in the dead reckoning mode before the errors become larger than specified.

2.2 Target Tracking Kalman Filter

The target tracking Kalman filter outlined in this section makes use of the DRKF vessel velocity and position estimates and radar measurements in tracking a selected radar target. A first order Gauss-Markov process is also used in this filter for both the vessel and target acceleration models. Since speedlog measurements are not used in this filter, it is not necessary to have states which estimate ocean current dynamics.

System Model

The following states are defined for TTKF

States to be estimated

x_1 = *vessel x position*
x_2 = *vessel y position*
x_3 = *vessel velocity over ground in x direction*
x_4 = *vessel velocity over ground in y direction*
x_5 = *vessel acceleration over ground in x direction*
x_6 = *vessel acceleration over ground in y direction*
x_7 = *target x position*
x_8 = *target y position* (8a)
x_9 = *target velocity over ground in x direction*
x_{10} = *target velocity over ground in y direction*
x_{11} = *target acceleration over ground in x direction*
x_{12} = *target acceleration over ground in y direction*

The input transition matrix, G, for the continuous target tracking system is defined as

$$G^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \alpha_v & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_v & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_t \end{bmatrix} \quad (8b)$$

where α_v and α_t are defined by equation (1) *for the vessel and target, respectively.*

The discrete transition matrix, Φ, is defined as

$$\Phi = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_v & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_v & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \Delta t & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_t \end{bmatrix} \quad (8c)$$

where a_v and a_t are defined by equation (2) *for the vessel and target, respectively.*

The discrete covariance structure, Q_k, of the state input sequence $\underline{w}(k)$ is calculated as outlined in equation (6).

Measurement System

The measurements presently incorporated into the target tracking filter consist of the Kalman Filtered position and velocity estimates from the DRKF and the radar range and azimuth to the target, i.e.,

Measurements

z_1 = *DRKF x position*
z_2 = *DRKF y position* (9a)
z_3 = *DRKF speed over ground in x direction*
x_4 = *DRKF speed over ground in y direction*
z_5 = *Radar range to target*
z_6 = *Radar azimuth angle to target*

The linearized measurement matrix, H, is defined as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -H_{51} & -H_{52} & 0 & 0 & 0 & 0 & H_{51} & H_{52} & 0 & 0 & 0 & 0 \\ -H_{62} & H_{61} & 0 & 0 & 0 & 0 & H_{62} & -H_{61} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9b)$$

$$\text{where } H_{51} = \frac{(x_7 - x_1)}{\sqrt{(x_7 - x_1)^2 + (x_8 - x_2)^2}},$$

$$H_{52} = \frac{(x_8 - x_2)}{\sqrt{(x_7 - x_1)^2 + (x_8 - x_2)^2}}, \tag{9c}$$

$$H_{61} = \frac{(x_7 - x_1)}{(x_7 - x_1)^2 + (x_8 - x_2)^2} \text{ and}$$

$$H_{62} = \frac{(x_8 - x_2)}{(x_7 - x_1)^2 + (x_8 - x_2)^2}$$

2.3 Dead Reckoning/Target Tracking Kalman Filter

The suboptimal dead reckoning/target tracking Kalman Filter (DRTTKF) filter, illustrated in Figure 3, is a combination of the two previously outlined filters. The DRTTKF consists of the eleven state DRKF serially providing the vessel's position and velocity estimates to the multiple twelve state TTKFs. This results in decoupling the vessel and target kinematic states in order to reduce computer computation. The percentage in computational saving by using the suboptimal DRTTKF is given by

$$\left(1 - \frac{(11^3 + n \times 12^3)}{(11 + 6 \times n)^3}\right) \times 100 \tag{10}$$

$$\text{where } n = \text{number of targets tracked}$$

In addition to the computer computational savings, it should also be noted that generally vessel dynamic measurements are obtained every second while it takes two seconds for one complete radar sweep. Thus target positional and velocity estimates cannot occur at the same rate as those of the vessel and are not readily integrated into the same system model as the DRKF.

Table III illustrates the target's kinematics that were simulated. The vessel test column corresponds to Test 1 and 3 outlined in Tables I and II. Figure 4 illustrates the estimated and true x position values for the three test outlined in Table III. Figure 5 illustrates the corresponding estimation error covariance.

The errors in vessel position and velocity estimate in the DRKF are fed to the TTKF measurement error covariance matrix; thus the TTKF vessel measurements are assumed white, gaussian, and time variant. This degrades the TTKF filter error estimates somewhat, because elements of the DRKF error covariance matrix are correlated and nonwhite. This is evident from Figure 5. Figure 5 shows that in the dead reckoning mode in the DRKF there is no corresponding rise in the error estimate in the TTKF.

As outlined in the DRKF results, Figure 5 illustrates that the higher the velocity vector and bearing rate, the greater the error estimate. The oscillations in the error estimation reflects the variations in the partials in the linearized matrix, H. The slightly higher error estimates in Test 2 compared to that of Test 1 reflect the higher variance and lower time constant in the acceleration models. The greater fluctuation in the error in Test 1 compared to that of Test 2 and 3 reflects the fluctuations in error of Test 1 compared to Test 3 in the DRKF results.

Figure 4 shows that the difference between the estimated and target UTM x position values is quite small, except when in "bottom-lock" mode.

Table III. Target Kinematic Values*

Test		Tgt/Ves Acceleration		Tgt Velocity		Tgt Bearing Rate
Tgt	Ves	$\sigma_{t,v}^2$ m^2/s^4	$\beta_{t,v}$ 1/s	V_{xo} m/s	V_{yo} m/s	deg/s
1	1	0.005	0.1	5	5	4.58
2	3	0.001	0.07	5	5	4.58
3	3	0.001	0.07	10	10	6.89

*Tgt and Ves are abbreviations for the target and vessel, respectively. The initial non-zero velocity vector occurs at a time of 40 secs, while the bearing rate occurs at 80 secs from time of simulation. The white noise measurement errors are 10 metres for range and 1 deg for bearing.

Figure 3. Schematic of TTKF Configuration

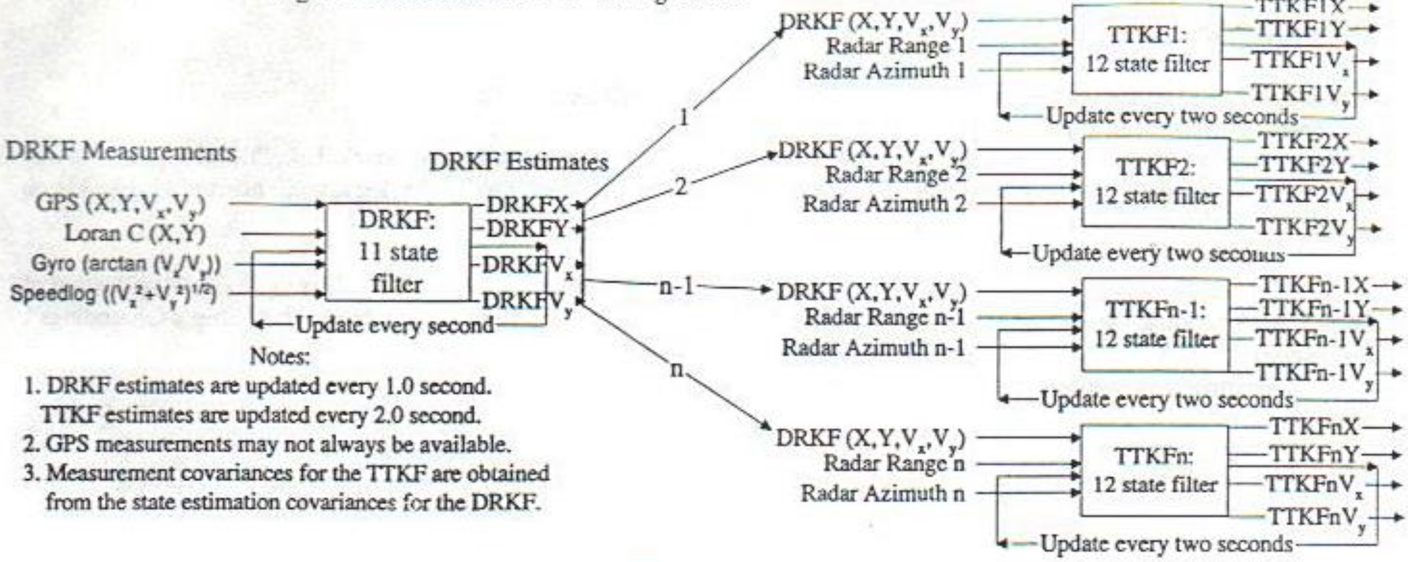


Figure 4. Comparing Estimated and True Target UTM X Values

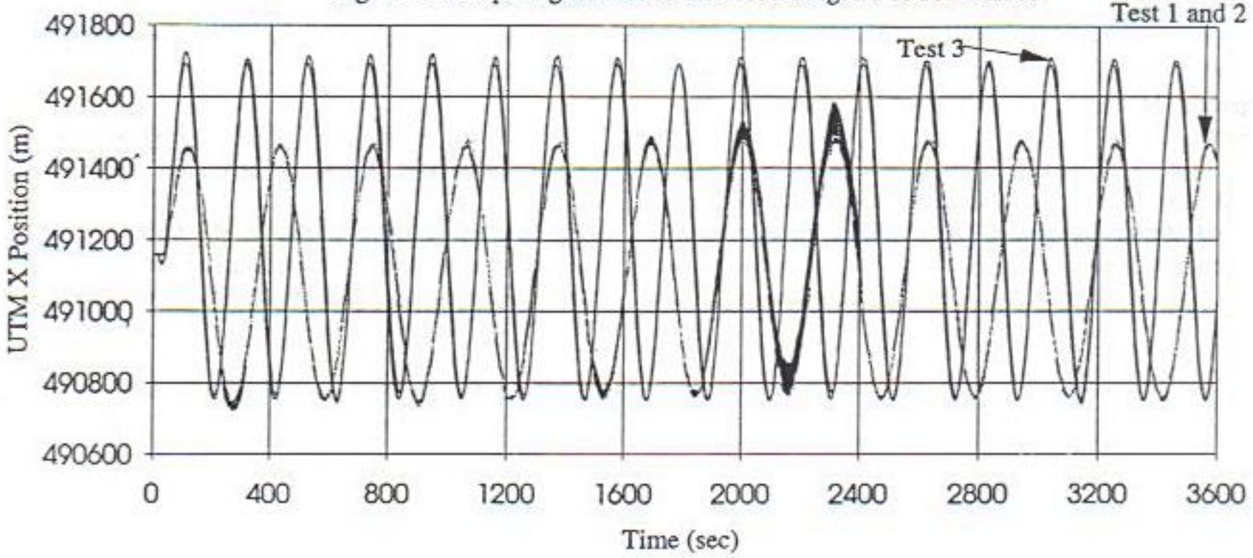
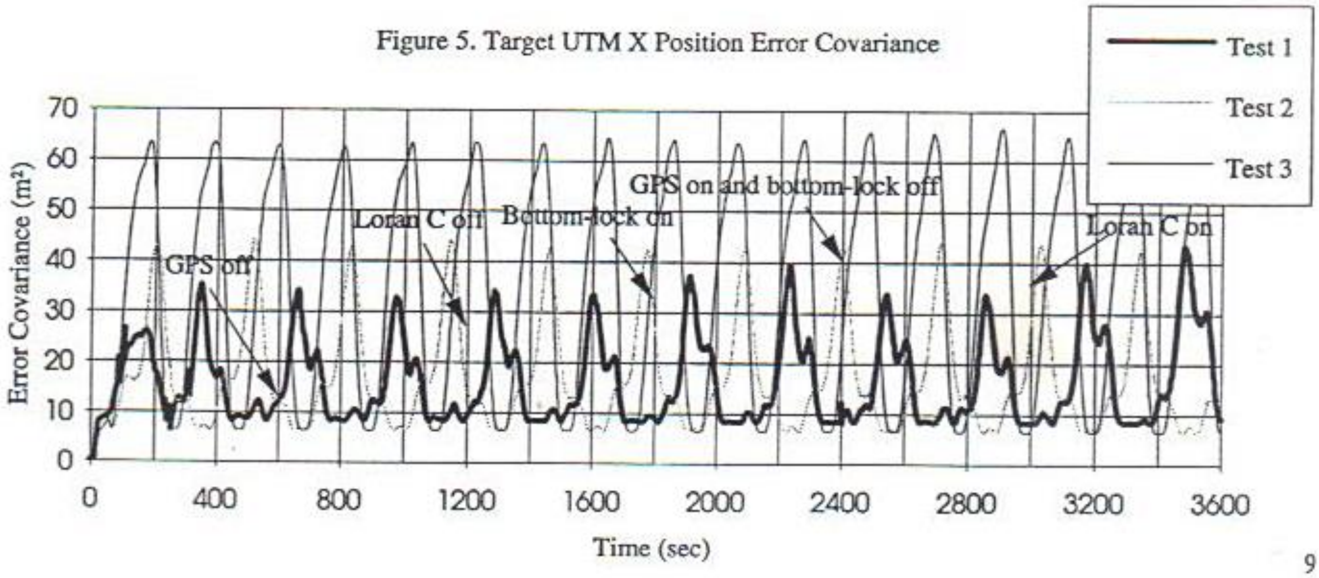


Figure 5. Target UTM X Position Error Covariance



CONCLUSIONS

A decoupled dead reckoning/target tracking Kalman Filter has been developed and found to perform close to the coupled system, resulting in a substantial savings of computer computation. The cost of decoupling the system model was a slight degradation in the target's estimated error covariance.

Further work is going into the application of radar overlay and Kalman Filters in an ECDIS system in order that mariner navigation is made more reliable, accurate, and safe.

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