ECDIS[1] Steering Algorithm for Vessel Autopilot

Erick Baziw, P.Eng.

Senior Systems Software Engineer
Offshore Systems, Ltd.
North Vancouver, B.C. Canada

Abstract - Offshore Systems Ltd. (OSL) produces an ECDIS[1], which is called the Electronic Chart Precise Integrated Navigation System (**ECPINS®**). The objective of the ECPINS® research is to incorporate the essentials of marine navigation into a single bridge display that automates marine navigation, thereby reducing cost and increasing safety.

Currently OSL has incorporated electronic charts, positioning algorithms, route planning, and radar overlay into its ECDIS. The positioning algorithms implement Kalman Filters to track radar targets and to estimate the vessel's position. This paper presents a proposed steering algorithm based on optimal control concepts for the vessel's autopilot.

The performance of the steering algorithm for the vessel's autopilot is evaluated by processing synthetic data. Desired route and vessel steering trajectories are presented and the performance is evaluated. From the results obtained and analyses conducted, it was found that this preliminary autopilot design provided a good basis for a final steering algorithm.

1.0 INTRODUCTION

The most important feature of an ECDIS system is that it accurately display a vessel's postion and velocity vector on an electronic navigational chart (ENC) in real-time. OSL's engineering department has developed considerable experience in the application of Kalman Filters([1] and [2]) in the estimation of a vessel's kinematics from noisey and nonlinear measurement data. The next logical step from estimating a vessel's kinematics is to control the vessel's steering mechanism in order to drive the vessel's states to desired values. This might be required, for example, during route surveying, docking, or navigation in critical channels.

There are several ways in which one could specify a control problem for the purpose of designing an autopilot for the vessel. It was decided to start with a relatively simple control problem so that insight into a practical solution could be obtained. A relatively simple control problem can be specified if it is assumed that one is only concerned with controlling the transverse (crosstrack) position error and the heading angle error for a given route; it is further assumed that these errors should be driven to zero in minimum time.

The first step in designing the control strategy is to adequately define the physical problem. It is important that the physical problem be defined by a set of easily solved differential equations. Section 2.0 outlines the physical model and the differential equation defining the vessel's dynamics when controlling the vessel's center of gravity (cg).

The next step in designing the vessel autopilot problem is to determine which control theory is most applicable to solving the desired boundary conditions. The control strategy outlined here relies on certain results from the Maximum Principle of Pontryagin. Section 2.1 outlines the governing control equations and the derivation of the bang-bang switching times of the rudder control.

In Section 2.2 the performance of the steering algorithm is evaluated by processing synthetic data derived from a vessel travelling along a predefined route, with various environmental conditions. Crosstrack and yaw error data are presented along with the controller's rudder commands. In addition, Section 2.2 discusses the control strategy for dead band modes of operation.

2.0 PHYSICAL PROBLEM

The simplified vessel dynamics are specified by applying Nomoto's First-Order Model relating a vessel yaw rate to the rudder angle ([3] and [4]). The model is specified by equation (1a), where it is assumed that the coupling between the linear yaw and sway are small.

$$T\ddot{\psi} + \dot{\psi} = K_{\psi}\sin\delta \qquad (1a)$$

where

ψ = *vessel's yaw angle in radians*
δ = *vessel's rudder angle in radians*

T = time constant relating how long it takes the vessel turning rate to respond to angle (δ).

K_{ψ} = constant (or slowly varying function) which is dependent upon the propeller thrust (T), the vessel turning drag (D_{ψ}), and the moment arm from the center of gravity to the rudder (l), ie., $K_{\psi} = fcn(T, D_{\psi}, l)$

The simplified expression of the vessel's velocity magnitude (V_o) is defined as

[1] Electronic Chart Display and Information System

$$V_o \equiv K_v * T \qquad \textit{vessel velocity} \qquad (1b)$$

In this equations K_v is a constant (or slowly varying function) which is dependent upon the propeller thrust (T) and the vessel velocity drag (D_v), ie.,

$$K_v = fcn(T, D_v) \qquad \textit{vessel velocity constant} \qquad (1c)$$

Figure 1 illustrates the vessel's kinematics.

It is assumed that the dynamics of the rudder are much quicker than the dynamics of the vessel's turning rate. In this case the rudder motion (δ) is assumed to be instantaneous compared to changes in the heading (ψ).

The coordinates for the vessel's motion are defined as follows:

X_N, Y_N are the fixed navigation coordinates
X_N', Y_N' are the desired route coordinates
where
X_N' is perpendicular to the desired route
Y_N' is along the desired route

These coordinate systems have a common origin, and ψ is measured clockwise from north (Y_N axis) as shown in Figure 1.

The position of the vessel is defined as follows:

x_N, y_N is the vessel location wrt the navigation coordinates
x_d, y_d is the desired vessel location wrt the navigation coordinates
x_N', y_N' is the vessel location wrt the desired route coordinates
x_d', y_d' is the desired location wrt the desired route coordinates
ψ is the heading wrt the navigation coordinates
ψ_d is the desired heading wrt the navigation coordinates
u_w', v_w' water current and/or wind velocity along X_N', Y_N' directions
$\dot{\psi}_w$ water current and/or wind velocity effecting yaw rate

The position, heading, and heading rate errors are defined as:

$$(\Delta x, \Delta y) = (x_N, y_N) - (x_d, y_d)$$
$$(\Delta x', \Delta y') = (x_N', y_N') - (x_d', y_d')$$
$$\Delta\psi = \psi - \psi_d$$
$$\Delta\dot{\psi} = \dot{\psi} - \dot{\psi}_d$$

The control problem is specified as controlling the transverse (crosstrack) position error ($\Delta x'$), the heading angle ($\Delta\psi$), and the heading rate ($\Delta\dot{\psi}$) such that they are driven to zero in minimum time. Of course, a different control strategy could be specified when it is important to control the translational position error ($\Delta y'$) to zero; this might be required, for example, for a portion of the vessel's route, or at the end of the vessel's route. This is a more complicated control problem and will be addressed in a separate paper. More specifically, we define the present control problem as follows:

given the initial states at time t^o,
$\Delta x'(t^o)$, $\Delta\psi(t^o)$, and $\Delta\dot{\psi}(t^o)$
determine a rudder strategy, $\delta(t)$, over the time interval $t^o \le t \le t^f$
such that the final states at time t^f,
$\Delta x'(t^f)$,$\Delta\psi(t^f)$, and $\Delta\dot{\psi}(t^f)$
are driven to zero in minimum time,
i.e., $t^f - t^o$ is minimized.

It is assumed that the rudder angle is limited as follows

$$|\delta(t)| \le \delta^{max}, \qquad (2a)$$

where δ^{max} is a specified constant, and δ^{max} could have different values depending on the particular steering mode.

For convenience a state vector $\underline{x}$ is defined as follows:

$$(x_1, x_2, x_3, x_4) \equiv (\Delta y', \Delta x', \Delta\psi, \Delta\dot{\psi}) \qquad (2b)$$

The differential equations can then be written as

$$\dot{x}_1 = V_o \cos x_3 + u_w' \qquad (3a)$$

$$\dot{x}_2 = V_o \sin x_3 + v_w' \qquad (3b)$$

$$\dot{x}_3 = x_4 + \dot{\psi}_w - \dot{\psi}_d \qquad (3c)$$

$$\dot{x}_4 = -\frac{1}{T}x_4 + \frac{K_\psi}{T}\sin\delta \qquad (3d)$$

As previously noted, u_w', v_w', $\dot{\psi}_w$ and $\dot{\psi}_d$ are assumed to be known functions of time.

2.1 CONTROL EQUATIONS

The Hamiltonian for this control problem is given by

$$H(P, x, \delta) = \textstyle\sum_{i=2,3,4} \dot{x}_i P_i + 1 \qquad (4a)$$

substituting values for $\dot{x}_2$, $\dot{x}_3$,and $\dot{x}_4$ gives

$$H = P_2\left(V_o sin x_3 + v_w'\right) + P_3(x_4 + \dot{\psi}_w - \dot{\psi}_d) \qquad (4b)$$
$$+ P_4\left(-\frac{1}{T}x_4 + \frac{K}{T}\sin\delta\right) + 1$$

The adjoint equations are given by the partials

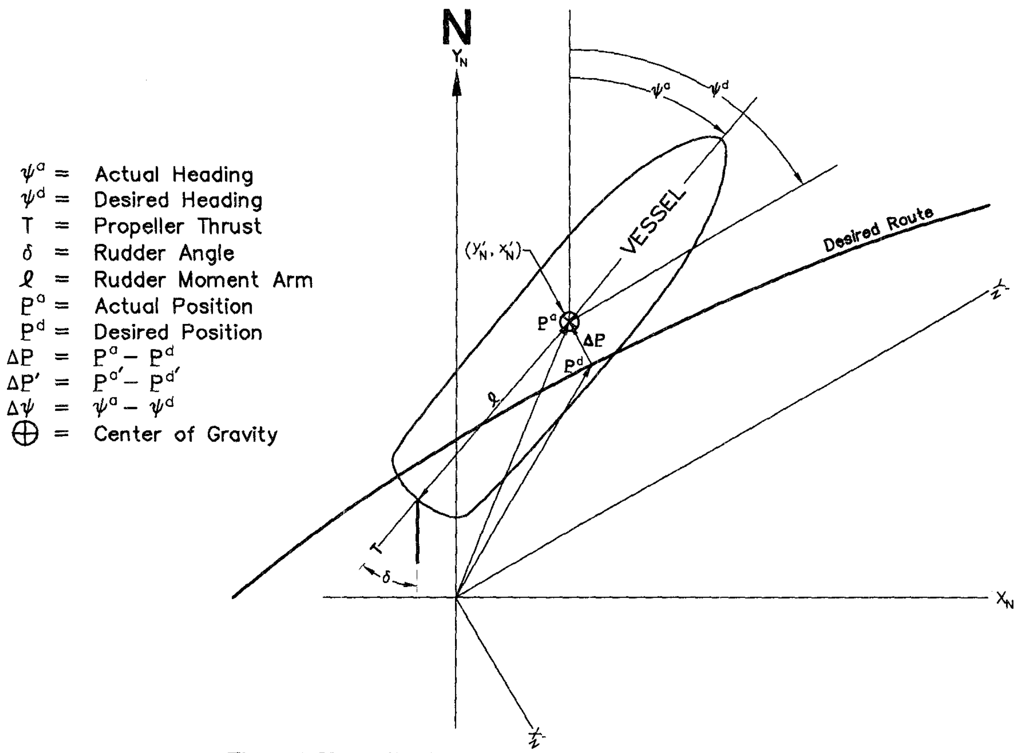


Figure 1. Vessel Configuration and Coordinates.

$$\dot{P}_i = \frac{\partial H}{\partial x_i}, \; i = 2,3,4.$$

thus

$$\dot{P}_2 = 0 \;\rightarrow P_2(t)=P_2^o, \; \textit{a constant}$$
$$\dot{P}_3 = -P_2^o V_o \cos x_3 \qquad (4c)$$
$$\dot{P}_4 = P_3 - \frac{1}{T}P_4$$

it is noted that

$$\ddot{P} + \frac{1}{T}\dot{P}_4 = -P_2^o V_o \cos x_3$$

If it is assumed that $|\delta| \leq 90°$, then to minimize H it is clear that

$$\delta(t) = -sign(P_4(t))\delta^{max} \qquad (4d)$$

where δ^{max} *is the maximum value of* $|\delta|$.

If we had an explicit solution for $P_4(t)$, the time optimal control strategy for δ would be specified. In general, the solution of P_4 in equation (4c) requires an iteration process on the initial values for P_4 (P_4^o), P_3 (P_3^o), and for P_2 (P_2^o) such that x_4, x_3, and x_2 at the end time (t^f) are zero (for example see [5]).

Another approach for evaluating $\delta(t)$ is to make the additional assumption concerning the number of switchings of $\delta(t)$ (ie., the number of zero crossings of the adjoint variable P_4). In particular, if we assume that x_3 does not get too large, so that we can approximate $\sin x_3$ by x_3, then we can apply certain results from the Maximum Principle of Pontryagin. In particular, Theorem 6-8 of [5] implies that a unique optimal control exists which will switch at most three times in the interval t^f - t^o. The optimal control strategy for a particular set of initial and final states can then be specified by the sign at t^o (Δ^o) and three time intervals (T_1, T_2, T_3), i.e., under these assumptions,

$$\delta(t) = fcn(K_\psi, \delta^{max}, \Delta^o, T_1, T_2, T_3)$$

In this case equation (3d) can be written as

$$\dot{x}_4 = -\frac{1}{T}x_4 + \frac{K_\psi'}{T}\Delta \qquad (4e)$$

where

$$K_\psi' = K_\psi \sin\delta_{max}$$

$$\Delta = \begin{cases} \Delta^o & \textit{for } t^o \leq t \leq t^o+T_1 \\ -\Delta^o & \textit{for } t^o+T_1 < t \leq t^o+T_1+T_2 \\ \Delta^o & \textit{for } t^o+T_1+T_2 < t \leq t^o+T_1+T_2+T_3 \end{cases} \qquad (4f)$$

The final values for x_2, x_3, x_4 can then be expressed as

$$x_4^f = x_4^o \exp^{-(T_1+T_2+T_3)/T} + I_1^\Delta \quad (4g)$$

$$x_3^f = x_3^o + I_1^w - T x_4^o \exp^{-(T_1+T_2+T_3)/T} + I_2^\Delta \quad (4h)$$

and

$$x_2^f = x_2^o + I_1^v + V_o \int_{T_1+T_2+T_3} x_3(t)dt \quad (4i)$$

$$\int x_3(t)dt = \int x_3^o + I_2^\psi - T x_4^o \int \exp^{t/T} dt + I_3^\Delta$$

The parameters in these equations are defined as follows:

$$\begin{aligned} I_1^\Delta &\equiv \frac{K_\psi'}{T} \exp^{-(T_1+T_2+T_3)/T} \int_{T_1+T_2+T_3} \exp^{\frac{t}{T}} \Delta(t)dt \\ &= K_\psi' \Delta^o (1 - 2\exp^{-T_3/T} + 2\exp^{-(T_2+T_3)/T} \\ &\quad - \exp^{-(T_1+T_2+T_3)/T}) \end{aligned} \quad (5a)$$

$$\begin{aligned} I_2^\Delta &\equiv \int_{T_1+T_2+T_3} I_1^\Delta(t)dt = K_\psi' \Delta^o (-T + 2T exp^{-T_3/T} \\ &\quad - 2T exp^{-(T_2+T_3)/2} + T exp^{-(T_1+T_2+T_3)/T} + T_1 - T_2 + T_3) \end{aligned} \quad (5b)$$

$$\begin{aligned} I_3^\Delta &\equiv \int_{T_1+T_2+T_3} I_2^\Delta(t)dt = \\ & K_\psi' \Delta^o ((T_1^2 - T_2^2 + T_3^2)/2 + T_1(T_2+T_3) - T_2 T_3 \\ & - T(T_1 - T_2 + T_3) + T^2(1 - \exp^{-(T_1+T_2+T_3)/T} \\ & + 2\exp^{-(T_2+T_3)/T} - 2\exp^{-T_3/T}) \end{aligned} \quad (5c)$$

$$I_1^\psi \equiv \int_{T_1+T_2+T_3} (\psi_w - \psi_d)\, dt \quad (5d)$$

$$I_2^\psi \equiv \int_{T_1+T_2+T_3} \int (\psi_w - \psi_d) dt \quad (5e)$$

$$I_1^v \equiv \int_{T_1+T_2+T_3} v_w'\, dt \quad (5f)$$

Equations (5a) to (5c) can be written more concisely by making the following substitutions

Let

$$S_3 \equiv T_1+T_2+T_3,\ S_2 \equiv T_2+T_3,\ E_1 \equiv \exp^{-T_3/T},$$
$$E_2 \equiv \exp^{-S_2/T},\ \text{and}\ E_3 \equiv \exp^{-S_3/T}$$

Equations (5a) to (5c) can then be written as

$$I_1^\Delta = K_\psi' \Delta^o [1 - E_3 + 2E_2 - 2E_1]$$

$$I_2^\Delta = K_\psi' \Delta^o [(T_1 - T_2 + T_3) - T(1 - E_3 + 2E_2 - 2E_1)]$$

$$\begin{aligned} I_3^\Delta = K_\psi' \Delta^o [&(T_1^2 - T_2^2 + T_3^2)/2 + T_1(T_2+T_3) - T_2 T_3 \\ & - T(T_1 - T_2 + T_3) + T^2(1 - E_3 + 2E_2 - 2E_1)] \end{aligned}$$

Remark: It is recalled that the environmental inputs ψ_w, ψ_d, and v_w' are assumed to be known functions of time, so that the integrals I_1^ψ, I_2^ψ, and I_1^v can be evaluated numerically. It is also reasonable to assume that ψ_w and v_w' are available during the control interval $T_1+T_2+T_3$, then these estimates can be used for the next sampling time. The previous time interval $T_1+T_2+T_3$ minus the sampling interval Δt would be used for this integration.

As is recalled, it is desired to drive the states x_4, x_3, and x_2 to zero in minimum time. Using the above definitions, the end values x_4^f, x_3^f, and x_2^f are given by

$$g_1 \equiv x_4^f = x_4^o E_3 + I_1^\Delta \quad (6a)$$
$$g_2 \equiv x_3^f = c_3^o + x_4^o T(1 - E_3) + I_2^\Delta \quad (6b)$$
$$g_3 \equiv x_2^f = c_2^o + c_2^1 S_3 + V_o x_4^o T^2 E_3 + V_o I_3^\Delta \quad (6c)$$

where

$$c_3^o \equiv x_3^o + I_1^\psi$$
$$c_2^o \equiv x_2^o + V_o I_2^\psi + I_1^v - V_o T^2 x_4^o$$
$$c_2^1 \equiv V_o x_3^o + V_o x_4^o T$$

One method for solving the constraint equations (ie., $\underline{g}(\underline{T}; \underline{\Delta}^o, \underline{x}^o, \underline{x}^f) = \underline{0}$, $\underline{T} \equiv (T_1, T_2, T_3)^T$, $T_i \geq 0$ i = 1,2,3) is to apply the Newton iteration technique. The Netwon iteration technique for this problem can be stated as

$$\underline{T}^{i+1} = \underline{T}^i - [F^T F]^{-1} F^T \underline{g}(\underline{T}^i) \quad (7)$$

$$\text{where } F = \left[\frac{\partial g}{\partial \underline{T}}\right]_{\underline{T} = \underline{T}^i}$$

The Newton technique requires an initial guess for the switching times (T_1^o, T_2^o, T_3^o), and then iterating on this guess by applying equation (7). This iteration is repeated until the constraints calculated from the new estimates on the switching times converge to within some predefined tolerance.

The partials of $\underline{g} = (g_1, g_2, g_3)$ wrt $\underline{T} = (T_1, T_2, T_3)$ are given by

$$F \equiv \frac{\partial g}{\partial \underline{T}} \equiv [f_{ij}] \quad (8)$$

where

$$f_{11} = \frac{E_3}{T}(\Delta^{o'} - x_4^o)$$
$$f_{21} = \Delta^{o'}(1 - E_3) + c_3^i$$
$$f_{31} = V_o \Delta^{o'}(T[E_3 - 1] + S_3) + c_2^1 + c_2^i$$
$$f_{12} = [\Delta^{o'}(E_3 - 2E_2) - E_3 x_4^o]/T$$
$$f_{22} = \Delta^{o'}(2E_2 - E_3 - 1) + c_3^i$$
$$f_{32} = V_o \Delta^{o'}[T(1 + E_3 - 2E_2) + T_1 - T_2 - T_3] + c_2^1 + c_2^i$$
$$f_{13} = [\Delta^{o'}(1 - E_{123}) - E_3 x_4^o]/T$$
$$f_{23} = \Delta^{o'} E_{123} + c_3^i$$
$$f_{33} = V_o \Delta^{o'}(S_3^- - T E_{123}) + c_2^1 + c_2^i$$
$$c_3^i = \partial I_1^\psi / \partial T_i + E_3 x_4^o$$
$$c_2^i = V_o \partial I_2^\psi / \partial T_i + \partial I_1^v / \partial T_i - E_3 T V_o x_4^o$$

$$\Delta^{o'} = K'_\psi \Delta^o$$
$$E_{123} = 1 - E_3 + 2E_2 - 2E_1$$
$$S_3^- = T_1 - T_2 + T_3$$

The initial guess on the switching times is obtained by solving the control problem with the assumption that the time constant (T) is zero. This results in a second order problem.

Control Problem for T ≅ 0

For the case T = 0, differential equations (3a) to (3d) are written as

$$\dot{x}_1 = V_o \cos x_3 + u'_w \qquad (9a)$$
$$\dot{x}_2 = V_o \sin x_3 + v'_w \qquad (9b)$$
$$\dot{x}_3 = K_\psi \sin\delta + \dot{\psi}_w - \dot{\psi}_d \qquad (9c)$$

Theorem 6-8 of [5] implies that a unique optimal control exists which will switch at most two times in the interval $t^f - t^o$. The optimal control strategy for a particular set of initial and final states can then be specified by the sign at t^o (Δ^o) and two time intervals (T_1, T_2), i.e.,

$$\delta(t) = fcn(K_\psi, \delta^{max}, \Delta^o, T_1, T_2)$$

Under these assumptions, equation (9c) can be written as

$$\dot{x}_3 = K'_\psi \Delta + \dot{\psi}_w - \dot{\psi}_d \qquad (9d)$$

where

$$K'_\psi \equiv K_\psi \sin\delta^{max} \qquad (9e)$$

$$\Delta = \begin{cases} \Delta^o & \text{for } t^o \le t \le t^o + T_1 \\ -\Delta^o & \text{for } t^o + T_1 < t \le t^o + T_1 + T_2 \end{cases}$$

Thus

$$x_3^f = K'_\psi \Delta^o (T_1 - T_2) + \int_{T_1+T_2} (\dot{\psi}_w - \dot{\psi}_d) dt + x_3^o \qquad (9f)$$

$$x_2^f = V_o \int_{T_1+T_2} x_3(t) dt + \int_{T_1+T_2} v'_w(t) dt + x_2^o \qquad (9g)$$

As above, it is convenient to make the following definitions:

$$I_1^\Delta \equiv \int_{T_1+T_2} \Delta(t) dt = \Delta^o (T_1 - T_2) \qquad (9h)$$

$$I_2^\Delta \equiv \int_{T_1+T_2} \int \Delta(t) dt dt' = \frac{\Delta^o}{2}(T_1^2 + 2T_1T_2 - T_2^2) \qquad (9i)$$

where I_1^ψ, I_2^ψ, and I_1^v are defined in equations (5d) to (5f), but integrated only over interval $T_1 + T_2$.

Since $x_3^f = 0$, equation (9f) gives

$$\Delta^o (T_1 - T_2) = -\frac{(I_1^\psi + x_3^o)}{K'_\psi} \equiv -(I_1^{\psi'} + x_3^{o'})$$

$$T_2 = T_1 + x_3^{o''} \Delta^o, \quad \text{where} \begin{cases} x_3^{o''} \equiv I_1^{\psi'} + x_3^{o'} \\ I_1^{\psi'} \equiv I_1^\psi / K'_\psi \\ x_3^{o'} \equiv x_3^o / K'_\psi \end{cases} \qquad (9j)$$

Since $x_2^f = 0$, equation (9g) gives the following quadratic equation in T_1 (assuming $\sin x_3 \approx x_3$):

$$T_1^2 + 2\Delta^o x_3^{o'} T_1 + \left(\Delta^o \left(I_2^{\psi'} + I_1^{v'} + x_2^{o'} \right) - I_1^{\psi'} x_3^{o''} + \frac{x_3^{o'/2}}{2} \right) = 0 \qquad (9k)$$

$$\text{where } I_2^{\psi'} \equiv \frac{I_2^\psi}{K'_\psi}, \quad x_3^{o'} \equiv \frac{x_3^o}{K'_\psi}, \quad I_1^{v'} \equiv \frac{I_1^v}{V_o K'_\psi}, \quad x_2^{o'} \equiv \frac{x_2^o}{V_o K'_\psi}$$

The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$x_{1,2} = \frac{\left(-b \pm \sqrt{b^2 - 4ac}\right)}{2a},$$

$$\text{and } b^2 - 4ac = \begin{cases} > 0 & \text{real and unequal roots} \\ 0 & \text{real and equal roots} \\ < 0 & \text{imaginary roots} \end{cases}$$

From equation (9k) we obtain

$$R = b^2 - 4ac = 4x_3^{o/2} - 4\left(\Delta^o \left(I_2^{\Psi'} + I_1^{v'} + x_2^{o'} \right) + \frac{x_3^{o/2}}{2} - \frac{I_1^{\psi/2}}{2} \right) \qquad (9l)$$

The two roots from the quadratic equation (9k) are given by

$$T_1^+ = -\Delta^o x_3^{o'} + \sqrt{R} \quad \text{and } T_2^+ = T_1^+ + \Delta^o x_3^{o'}$$
$$T_1^- = -\Delta^o x_3^{o'} - \sqrt{R} \quad \text{and } T_2^- = T_1^- + \Delta^o x_3^{o'}$$

where R is defined by equation (9l).

The value selected for T_1 and T_2 will then be such that

$$T_1 + T_2 = \min_{i=+,-}(T_1^i + T_2^i), \quad \text{with } T_1^i, T_2^i \ge 0$$

From initial simulation results, it appears that for a vessel with $T \le 3$ seconds, the second order control equations provide adequate solutions. Generally, a vessel's T value is determined from the following equation

$$T = LT'/V_o,$$

where L is the length of the vessel, T' is the ratio of the yaw inertia coefficient to the yaw damping coefficient (≈ 0.2 to 2.0) (note: this parameter is obtained from graphs [4]), and V_o is the vessel velocity. For example, the second order solution would be appropriate for a vessel defined with $T' \approx 1$, $V_o \geq 5$m/sec, and $L \leq 15$m.

It should also be noted that the K_ψ value is determined from the following equation

$$K_\psi = K_\psi' V_o / L$$

where K_ψ' is the ratio of the turning moment coefficient to the yaw damping coefficient. A large K_ψ'/T' ratio is indicative of a vessel with good maneuverability [4].

The following algorithm is used for computing the control parameters Δ^o, T_1, T_2, and T_3 as a function of the initial states x_2^o, x_3^o, and x_4^o.

Initialize $T_1 = T_2 = 10^5$, $T_3 = 1$

Solve Second Order Control Problem (ie., $T \leq 3$)

1. Select $\Delta^o = +1$, then compute $R = (b^2 - 4ac)/4$

   Note: $$R = \left[-\Delta^o \left(I_2^{\psi'} + I_1^{v'} + x_2^{o'} \right) + \frac{x_3^{o'2}}{2} + \frac{I_1^{\psi'2}}{2} \right]$$

   if $R < 0$, go to 4.

2. Compute $T_1^{1,+} = -x_3^{o'} + \sqrt{R}$
   if $T_1^{1,+} < 0$, go to 3.
   $T_1 = T_1^{1,+}$ and $T_2^{1,+} = T_1^{1,+} + \Delta^o x_3^{o''}$
   if $T_2^{1,+} < 0$, go to 3. Otherwise $T_2 = T_2^{1,+}$.

3. Compute $T_1^{2,+} = -x_3^{o'} - \sqrt{R}$
   if $T_1^{2,+} < 0$, go to 4.
   $T_1^2 = T_1^{2,+}$ and $T_2^{2,+} = T_1^{2,+} + \Delta^o x_3^{o''}$
   if $T_2^{2,+} < 0$, go to 4. Otherwise $T_2^2 = T_2^{2,+}$.
   if $T_1^2 + T_2^2 < T_1 + T_2$, set $T_1 = T_1^2$ and $T_2 = T_2^2$.

4. Select $\Delta^o = -1$, then compute $R = (b^2 - 4ac)/4$
   if $R < 0$, go to 5.
   Compute $T_1^{1,-} = -x_3^{o'} - \sqrt{R}$
   if $T_1^{1,-} < 0$, go to 5.
   Otherwise $T_1^1 = T_1^{1,-}$ and $T_2^{1,-} = T_1^{1,-} + \Delta^o x_3^{o''}$
   if $T_2^{1,-} < 0$, go to 5. Otherwise $T_2^1 = T_2^{1,-}$
   if $T_1^1 + T_2^1 < T_1 + T_2$, set $T_1 = T_1^1$ and $T_2 = T_2^1$.

5. Compute $T_1^{2,-} = -x_3^{o'} - \sqrt{R}$
   if $T_1^{2,-} < 0$, go to 6.
   $T_1^2 = T_1^{2,-}$ and $T_2^{2,-} = T_1^{2,-} + \Delta^o x_3^{o''}$
   if $T_2^{2,-} < 0$, go to 6. Otherwise $T_2^2 = T_2^{2,-}$.
   if $T_1^2 + T_2^2 < T_1 + T_2$, set $T_1 = T_1^2$ and $T_2 = T_2^2$.

6. Check for Dead Bands
   if $|T_1| \leq T_1^{min}$, set $T_1 = 0$ and $\Delta^o = -\Delta^o$
   if $|T_1| \leq T_1^{min}$ and $|T_2| \leq T_2^{min}$, set $T_1 = 0$, $T_2 = 0$, and $\Delta^o = 0$.
   Note : T_1^{min}, T_2^{min}, and δ^{max} should be obtained from an algorithm that is a function of $|x_2^o|$ and $|x_3^o|$.

7. Compute the integrals $I_1^{\psi'}$, $I_2^{\psi'}$, $I_2^{v'}$ using the time interval $T_1 + T_2 - \Delta_3$. (Δ_3 = control computational interval)

Solve for Control Problem where $T > 3$

8. Initial guess $T^o = (T_1^o, T_2^o, 1)$ and Δ^o, where T_1^o, T_2^o, and Δ^o are solutions to the second order solution.

9. Evaluate [F] defined by equation (8), and $[F^T F]^{-1}$.

10. Apply Newton's iteration method until $\underline{g}(\underline{T}^i) < \Delta G$, where ΔG is a predefined tolerance.

11a. If $T_1 < 0$, then set $\Delta^o = -\Delta^o$,
Else
11b. Compute the integrals $I_1^{\psi'}$, $I_2^{\psi'}$, $I_2^{v'}$ using the time interval $T_1 + T_2 + T_3 - \Delta_3$.

12. Check for Dead Bands.

EXIT.

A block diagram of the suboptimal feedback control scheme for vessel steering is illustrated in Figure 2.

2.2 AUTOPILOT SIMULATION RESULTS

The performance of the steering algorithm is evaluated by processing synthetic data derived from a vessel travelling along a predefined route for these test cases. The first two test cases simulate a vessel with a small Nomoto time constant; therefore the second order control solution is applied.

Test cases 1 and 2 have a Nomoto First Order time constant of 3 seconds and a $K_\psi = 0.0349$ (1/s) (calculated for $\delta^{max} = 30°$, $V_o = 10$ knots, and $\dot{\psi}_{max} = 1°$/s). Test case 3 has a Nomoto time constant of 20 seconds with the same K_ψ as that for test cases 1 and 2.

The steering algorithm not only maintains positional and yaw accuracy of a vessel on its current leg, but it also maneuvers a vessel when it switches to a new leg. This maneuver is referred to as the "autopilot turning circle". Figure 3 illustrates the necessary variables to consider when calculating the "autopilot turning circle". The location on the current leg where the vessel switches to a new leg is determined by calculating a critical distance from the end point of the current leg. This critical distance is calculated as

R* ≡ *Turning Radius*, *D* ≡ *Critical Distance

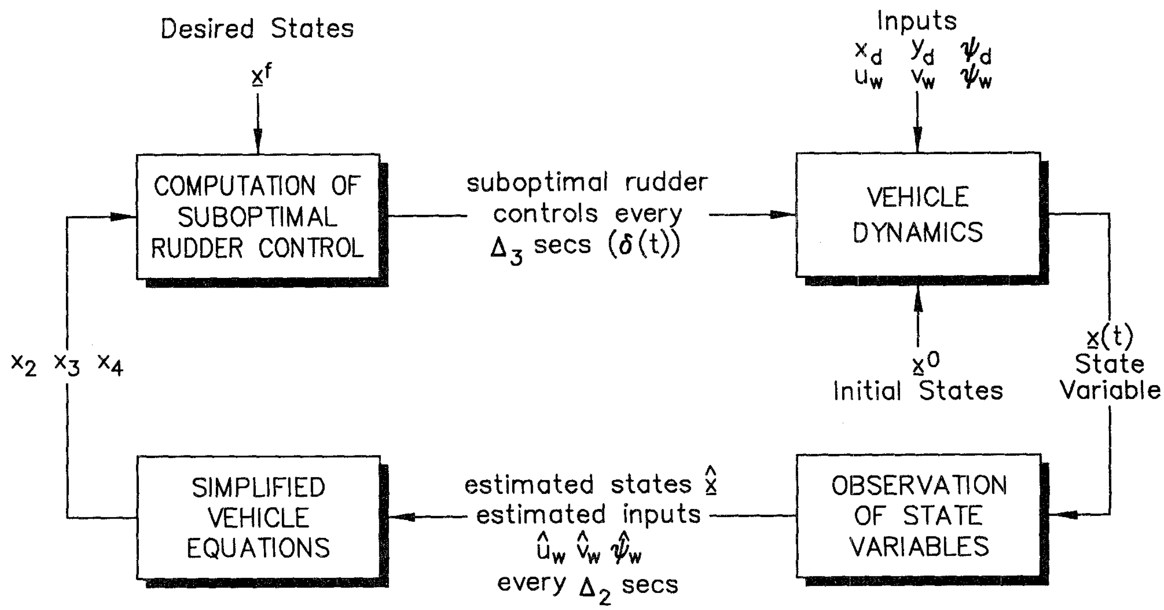
$$R = \frac{V_o}{|\dot{\psi}_d'|}, \quad \theta = (\alpha_1 - \alpha_2)/2, \quad \tan\theta = \frac{R}{D}$$

$$\therefore \quad D = \frac{R}{\tan\theta} \qquad (10)$$

Note: effects from v_w' are also taken into account.

$$\dot{\psi}_d' = 2\theta / [(2\theta/\dot{\psi}_d) + T]$$

The steering algorithm sets the autopilot into a dead band mode of operation when the switching times become less than



Notes:

1. State is measured every Δ_1 seconds. State is estimated every Δ_2 seconds. The rudder angle is computed, for the interval $T_1 + T_2 + T_3$, every Δ_3 seconds with $\Delta_1 \leq \Delta_2 \leq \Delta_3$.
2. The rudder angle $\delta(t)$ calculation is based on simplifying assumptions about the vehicle dynamics and the wind/water forces.
3. Uncertainties in the vehicle dynamics and inputs are accounted for by the feedback effect of recomputing $\delta(t)$ at a sufficiently high rate Δ_3.

Figure 2. Suboptimal Feedback Control Scheme

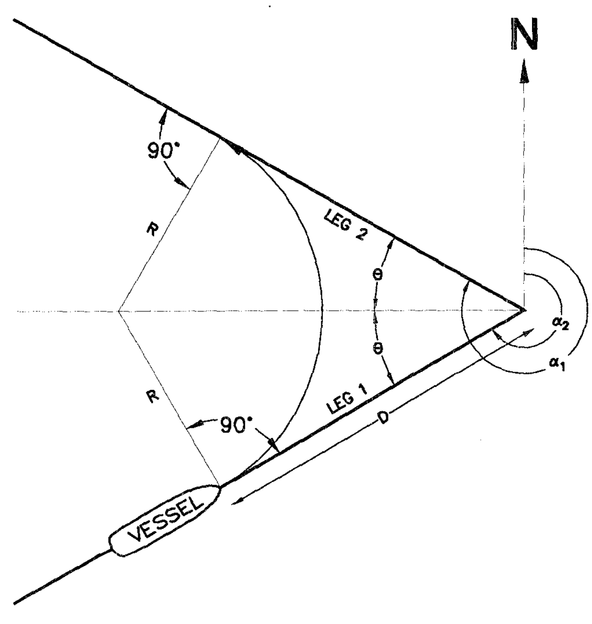


Figure 3. Autopilot Related Turning Circles

predefined T_1^{min}, T_2^{min}, and T_3^{min}. These minimum times are determined from the following algorithm

Assume $T \equiv 0$, *specify crosstrack and yaw error tolerances.* x^o_{2TOL} $\equiv$ *crosstrack error tolerance*

x^o_{3TOL} $\equiv$ *yaw error tolerance*

$$T_a^{min} = \frac{x^o_{3TOL}}{K_\psi \delta^{max}}, \qquad T_b^{min} = \frac{x^o_{2TOL}}{V_o}$$

$$if\ (T_a^{min} < T_b^{min}) \quad T^{min} = T_a^{min}$$

$$else \quad T^{min} = T_b^{min}$$

$$T_1^{min} = T_2^{min} = T_3^{min} = T^{min}$$

In addition to setting the autopilot into a dead band mode of operation, there are also limitations placed on δ^{max}. As the vessel approaches x_{2TOL}^o and x_{3TOL}^o, δ^{max} is exponentially decayed to some predefined $\delta^{max\prime}$ (ie., function of v_w', ψ_d, ψ_w, and V_o)

Figure 4a illustrates the test results for test case 1 where the positional and yaw time histories of a vessel, as it travels along a predefined route in Vancouver Harbour, is shown. The vessel route begins at leg 1 and progresses onto legs 2, 3, 4, 5, 6, 7, and 8, and then back again to leg 1. The vessel travels at a speed of 14 knots with no current effects. Figure 4a illustrates the origin of the vessel turning radius. Figure 4b shows both the yaw errors and rudder commands for test case 1. The large yaw errors are indicative of the manuever of the vessel when switching to a new leg and are not actual errors in the steering algorithm. The decay in δ^{max} (ie., $\delta^{max\prime}$) is pointed out along with the dead band modes of operation. Figure 4c illustrates the crosstrack errors of the vessel with respect to the active leg. The large crosstrack errors occur when the vessel maneuvers to a new leg and are also not actual errors in the steering algorithm.

Figure 5a illustrates test case 2 where the positional and yaw time history of a vessel as it travels along a predefined route in Vancouver Harbour is shown. As in test case 1, the vessel route begins at leg 1 and progresses onto legs 2, 3, 4, 5, 6, 7, and 8, and then back again to leg 1. The vessel travels at a speed of 14 knots with a 2 knot current @45° from true north. Figure 5a illustrates the origin of the vessel turning radius.

Figure 5b shows both the yaw errors and rudder commands for test case 2. As in case test 1, the large yaw errors are indicative of the manuever of the vessel when switching to a new leg. In this test there is also a decay in δ^{max} (ie., $\delta^{max\prime}$) and dead band modes of operation. Figure 5b also illustrates the yaw error offsets. These offsets are due to the steering algorithm counteracting the 2 knot current @45° from true north. Figure 5c illustrates the crosstrack errors of the vessel with respect to the active leg. As in test case 1, the large crosstrack errors occur when the vessel maneuvers to a new leg.

Figure 6a illustrates test case 3 which has identical vessel model parameters to that of test cases 1 and 2 except for the Nomoto Time Constant which is set to 20 seconds. The vessel is following the same route as the previous tests where it starts at leg 1 and progresses onto legs 2, 3, 4, 5, 6, 7, and 8, and then back again to leg 1. In this test the vessel travels at a speed of 14 knots with no current effects. Figure 6a illustrates the origin of the vessel turning radius.

Figure 6b illustrates the decay in δ^{max} (ie., $\delta^{max\prime}$) and the dead band modes of operation as in test cases 1 and 2. Figure 6c shows the crosstrack errors of the vessel with respect to the active leg. Figures 6b and 6c illustrate relatively more sluggish vessel dynamics for test case 3 compared to that of test cases 1 and 2. This is due to a larger Nomoto Time Constant and small K_ψ value.

3.0 CONCLUSIONS

A vessel steering algorithm has been presented where it is required to drive the transverse (crosstrack) position error and the heading angle error and rate to zero in minumum time. In addition, a control strategy has been outlined for the dead band modes of operation.

The performance of steering algorithm was evaluated by processing synthetic data. Crosstrack and yaw errors were analyzed along with with the controllers rudder commands. From the results obtained and analyses conducted, it was found that this preliminary autopiloit design provided a good basis for a final steering algorithm.

A more detailed discussion of the convergence properties of the iteration scheme employed in this paper, and of the numerical solution of the adjoint variables formulated will be presented in a future paper.

ACKNOWLEDGEMENTS

The Author expresses his thanks to Harvey Russell, Engineering Manager, for encouraging and reviewing this work, and to Helmut Lanziner and OSL for supporting this work. The Author would also like to thank John Baziw for his helpful suggestions.

REFERENCES

[1] Baziw, E.J., "Dead Reckoning/Target Tracking in an ECDIS Environment", presented at the Institute of Navigation national technical meeting, January 24-26, 1994.

[2] Baziw, E.J., "Field Trials of ECPINS® Vessel Positioning Algorithm", to be presented at the IEEE PLANS '96 technical meeting, April, 1996.

[3] Fossen, T.I, Guidance and Control of Ocean Vehicles, John Wiley and Sons Ltd., pp.172-176, 1994.

[4] Lewis, E.V., Principles of Naval Architecture Second Revision, Volume III Motions in Waves and Controllability, The Society of Naval Architects and Marine Engineers, pp. 205-208, 1989.

[5] Athans, A., and Falb, P.L., OPTIMAL CONTROL, McGraw-Hill Book Company, pp. 241-284, 402, 1966.

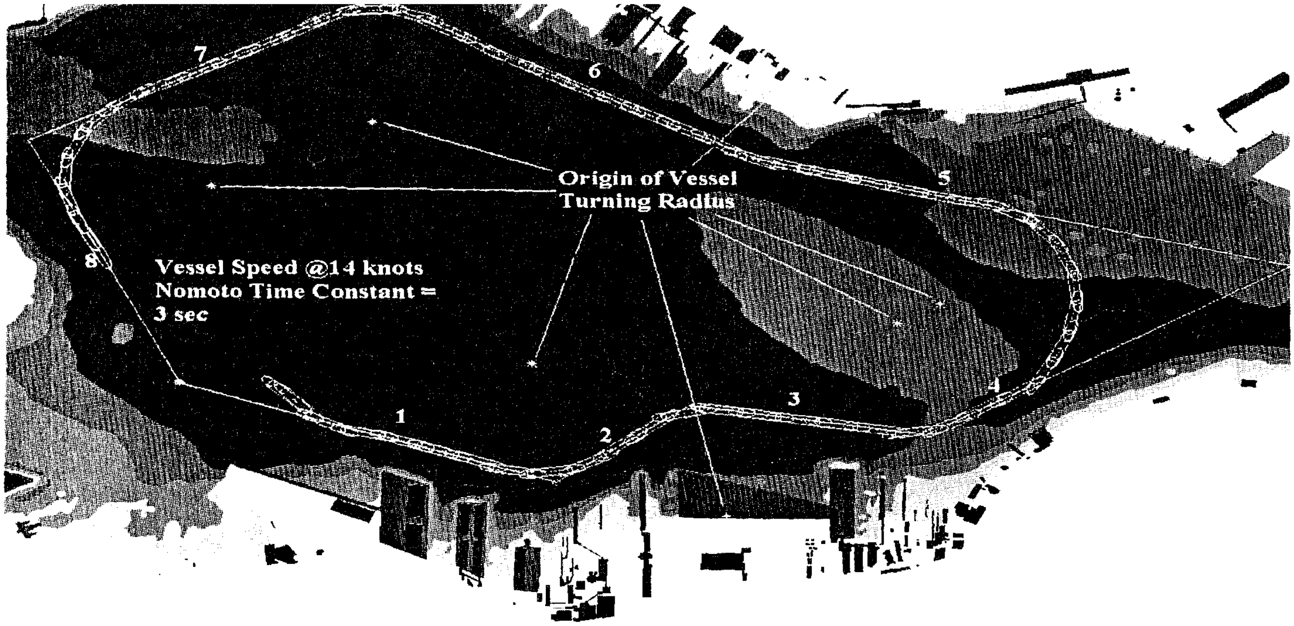


Figure 4a. Vessel Route for Test Case 1 (T = 3s, V_o = 14kt, K_ψ = 0.0349s^{-1}, and δ^{max} = 30°).

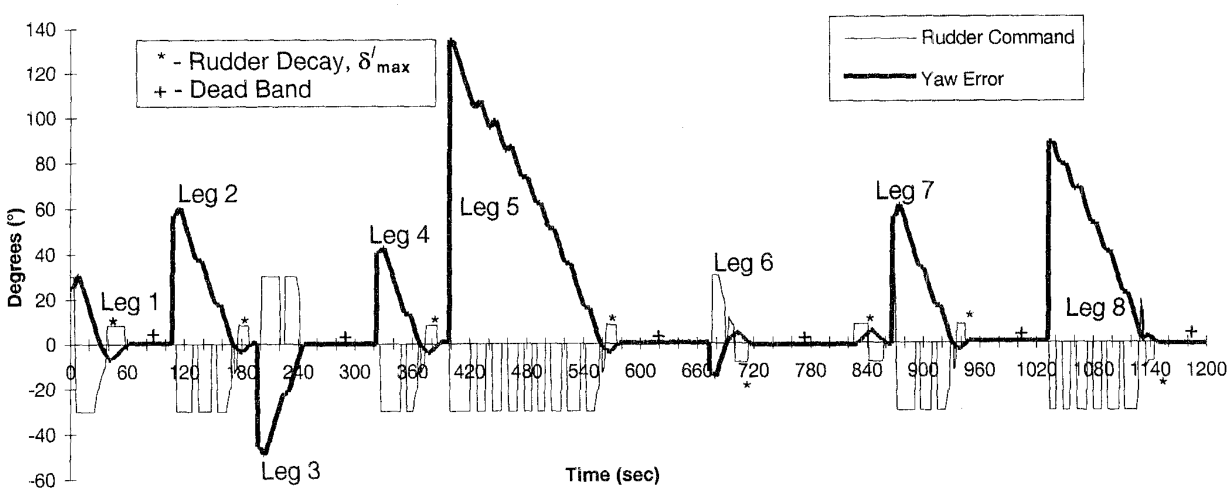


Figure 4b. Yaw Error and Rudder Command for Test Case 1.

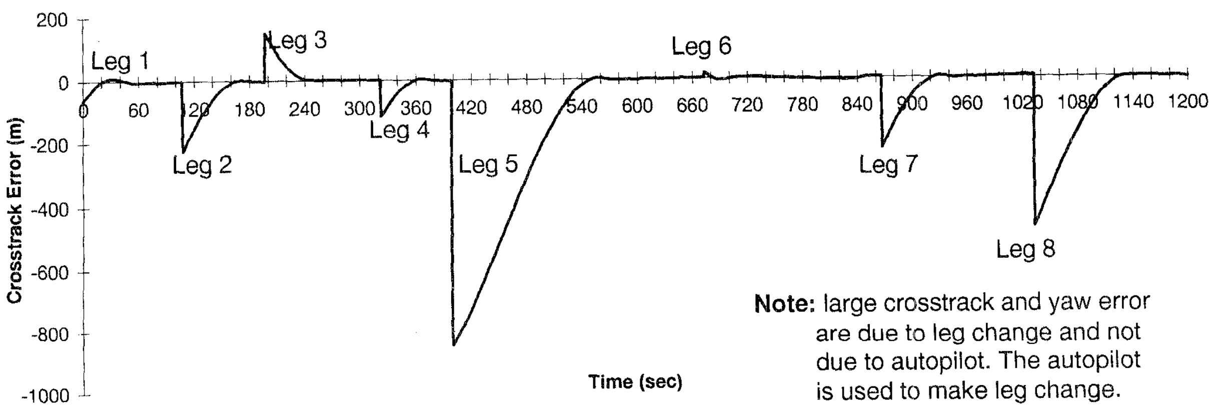


Figure 4c. Crosstrack Error for Test Case 1.

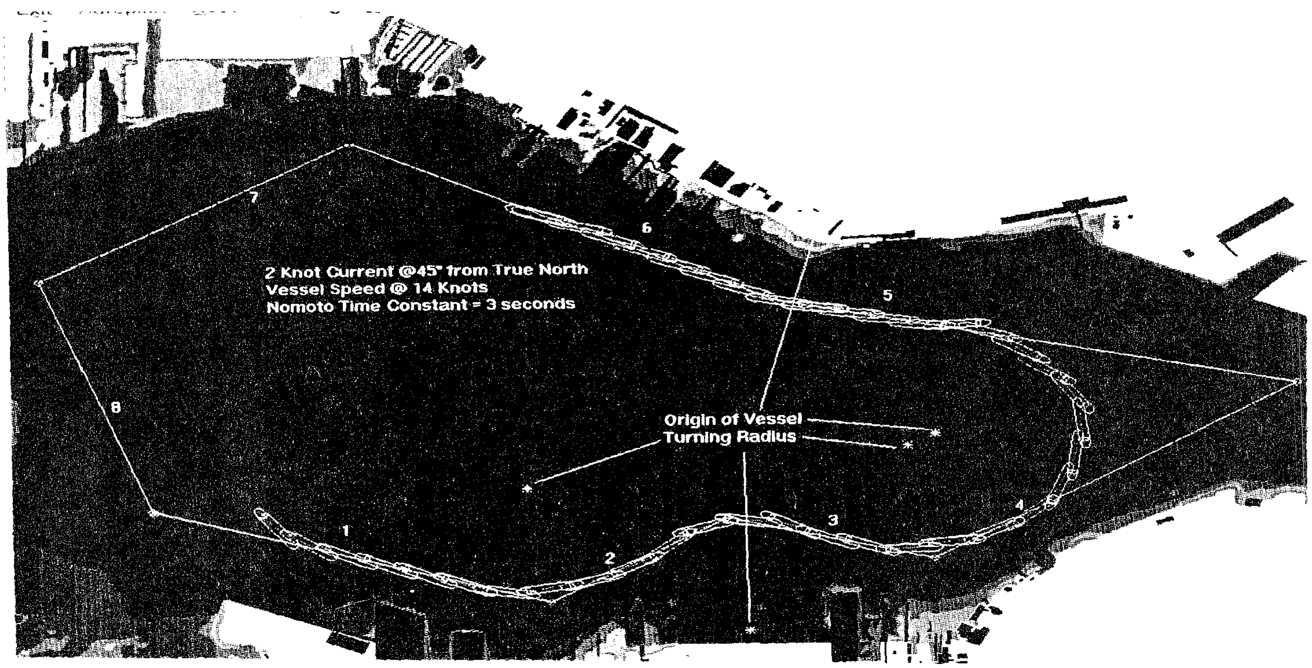


Figure 5a. Vessel Route for Test Case 2 (T = 3s, V_o = 14kt, K_V = 0.0349s^{-1}, and δ^{max} = 30°).

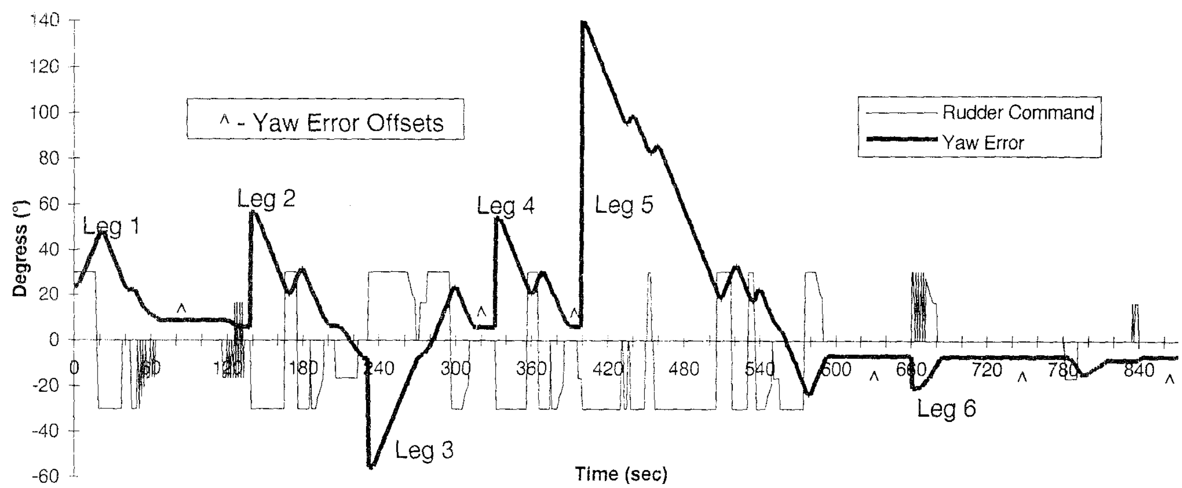


Figure 5b. Yaw Error and Rudder Command for Test Case 2.

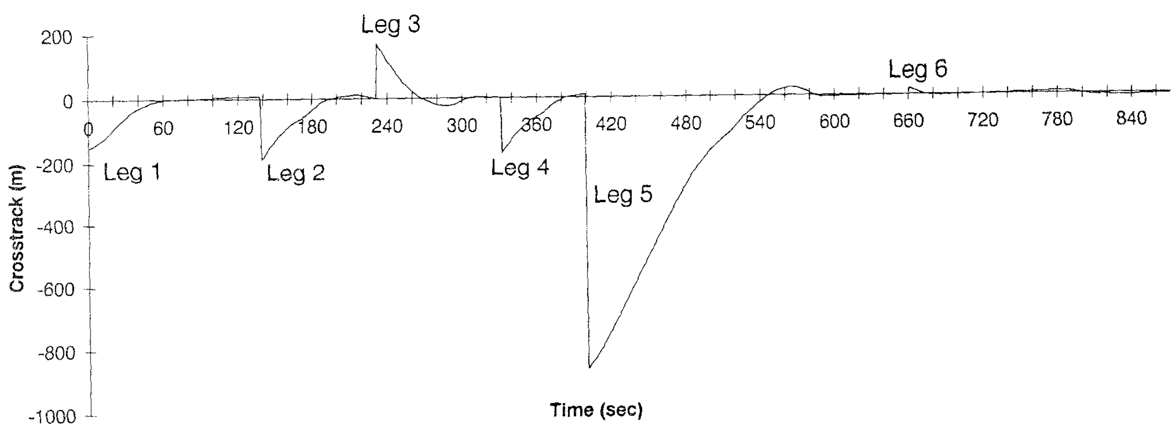


Figure 5c. Crosstrack Error for Test Case 2.

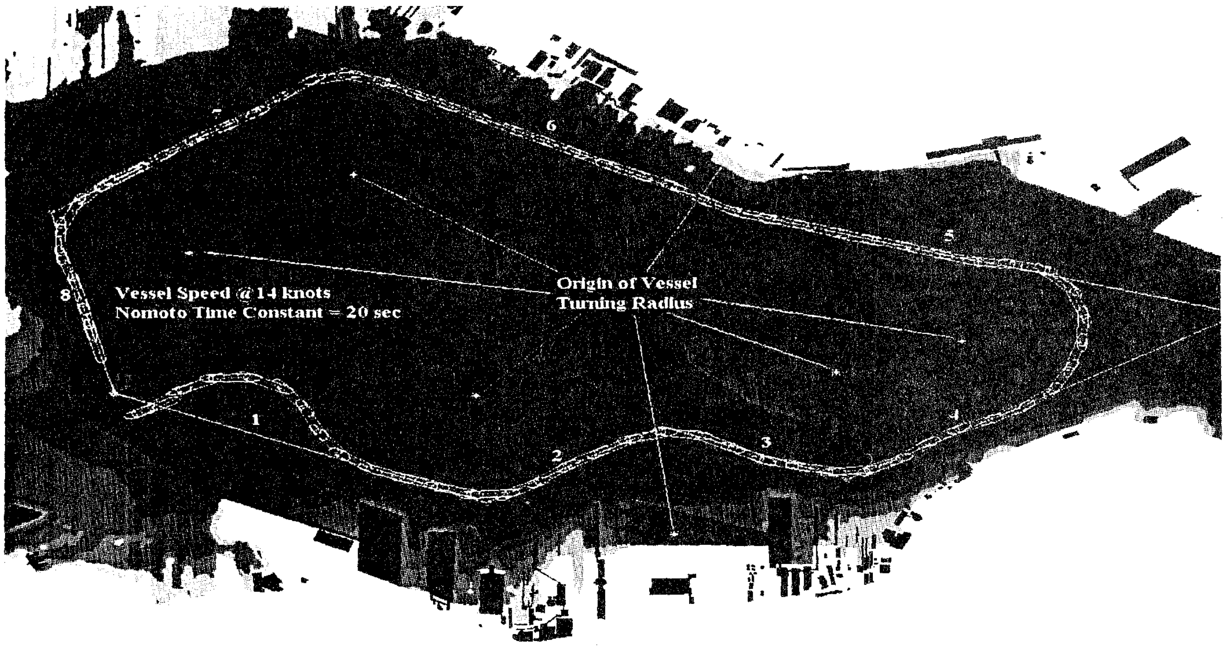


Figure 6a. Vessel Route for Test Case 3 (T = 20s, V_o = 14kt, K_ψ = 0.0349s^{-1}, and δ^{max} = 30°).

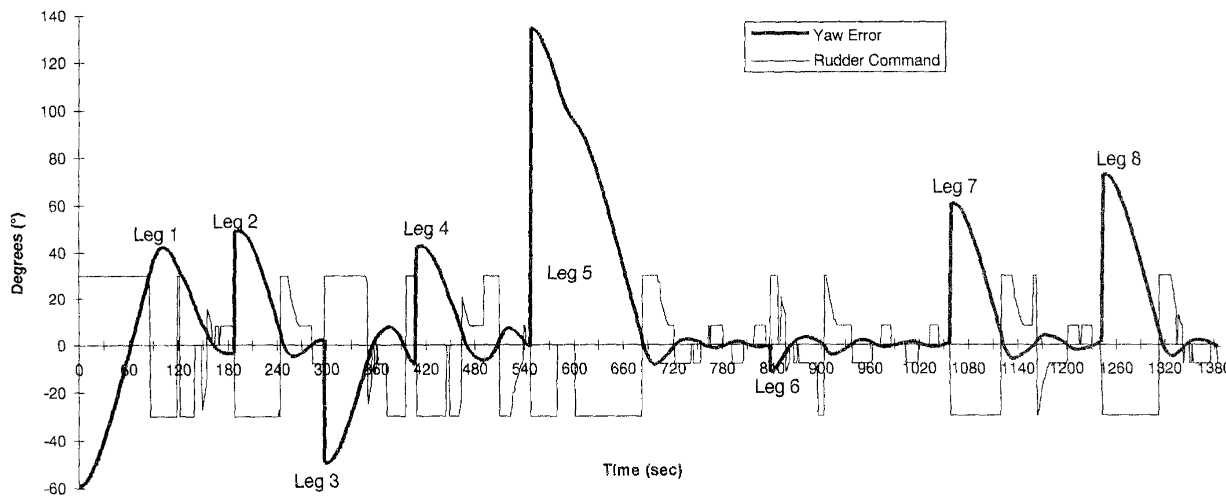


Figure 6b. Yaw Error and Rudder Command for Test Case 3.

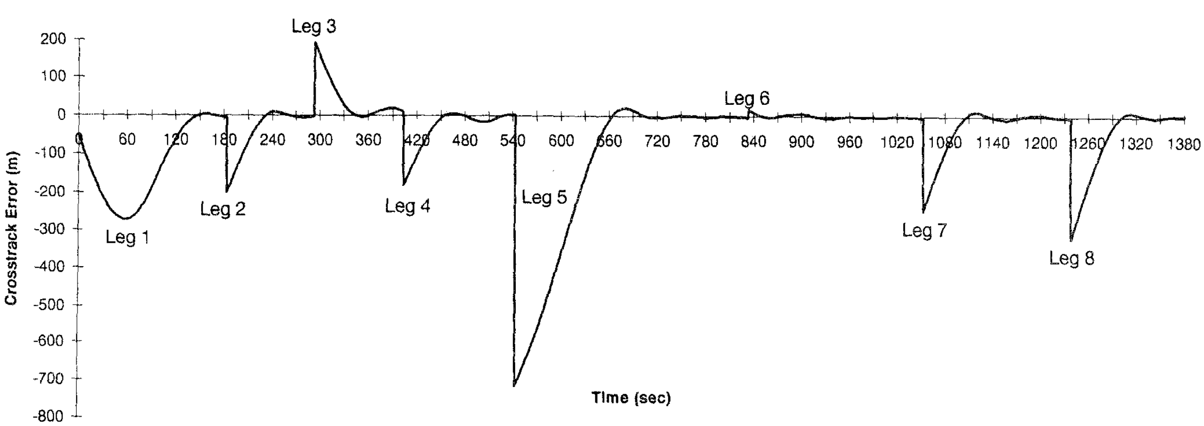


Figure 6c. Crosstrack Error for Test Case 3.